F. Cajori

A Soulié

B. Soulié

HAWNEY'S

COMPLETE MEASURER:

OR, THE

WHOLE ART OF MEASURING.

BEING

A PLAIN AND COMPREHENSIVE

TREATISE ON PRACTICAL GEOMETRY AND MENSURATION.

Corrected and improved by T. Keith.

THIRD EDITION.

WITH AN APPENDIX CONTAINING RULES AND EXAMPLES FOR FINDING THE WEIGHT AND DIMENSIONS OF BALLS AND SHELLS, WITH THEIR APPROPRIATE QUANTITIES OF POWDER.

REVISED AND CORRECTED BY JOHN D. CRAIG,
PROFESSOR OF MATHEMATICS.

BALTIMORE:
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1813.

English Work

District of Maryland, to wit:

SEAL. BE IT REMEMBERED, that on this first day of May, in the thirty-seventh year of the Independence of the United States of America, Fielding Lucas, junr. of the said district, hath deposited in this office, the title of a book the right whereof he claims as proprietor, in the words and figures following, to wit:

"Hawney's Complete Measurer: or, the whole art of measuring. Being a plain and comprehensive treatise on practical geometry and mensuration. Corrected and improved by T. Keith. Third edition, with an appendix containing rules and examples for finding the weight and dimensions of balls and shells, with their appropriate quantities of powder. Revised and corrected by John D. Craig, professor of mathematics."

In conformity to the act of the congress of the United States, intitled, "An act for the encouragement of learning by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned." And also to the act, intitled "An act supplementary to the act, intitled, 'An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned,' and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

PHILIP MOORE,
Clerk of the District of Maryland.

PREFACE

BY THE EDITOR.

THE many editions which Mr. Hawney's Treatise on Mensuration has gone through, are evident proofs of its general utility. But at the time when this work was first published, it was not the practice in schools for each scholar to have a printed book by him, while engaged in the study of arithmetic and mensuration; consequently Mr. Hawney gave no particular examples as exercises for a learner, without working them at full length, and explaining every step. Though this was perhaps originally an advantage to the book, it precluded the use of it in our modern schools; for with such assistance, a boy of good abilities would naturally become indolent, for want of something to exert his genius; and a boy of a heavy disposition would be induced to copy all his work from the book.

To remedy these inconveniences, the proprietors of the work engaged the present editor to make such alterations and additions as

would render it useful in schools, without diminishing its former plainness and perspicuity.

On examination of the sixteenth edition, the editor found it to be replete with errors, owing to the inattention, or incapacity of those who had the care of the press, since the death of the author. Hence arose the necessity of working every example anew, and examining every rule with the greatest care. This was attended with considerable labour, and at the same time was not a very agreeable task; for as the nature and plan of the work has undergone little or no change, so the merit is still due to the author, while the errors, if any of consequence are left, will be attached to the present editor.

The first nine chapters are in substance the same as in former editions. The Xth, XIth, and XIIth are added by the editor, besides various rules and observations throughout the work, the principal of which are distinguished in the table of contents by prefixing an asterisk.*

The mensuration of the five regular bodies is placed immediately after the mensuration of solids, and these are succeeded by board and timber measure.

In the former editions, timber measuring was divided into several sections; as, *squared timber*, or pieces of timber in the form of a parallelopipedon; *unequal squared timber*, or pieces of timber in the form of the frustum of a pyramid; *round timber with equal bases*, or pieces of timber in the form of a cylinder; *round timber with unequal bases*, or pieces of timber in the form of the frustum of a cone: and the contents were found, both by the *customary* method of measuring timber, and likewise by the rule for each respective solid. In this edition these examples are placed under the several rules to which they belong, and are distinguished by asterisks*; and in timber measuring they are brought forward, and solved by the *customary* method, with reference to the pages where they are truly solved. By this means several pages have been saved for the introduction of more useful matter.

The gauging and surveying, which were given in the former editions by way of appendix, are introduced in this edition without that distinction. The former has undergone such alterations as will render it very useful for those who are candidates for the excise, or who want to find the contents of different kinds of vessels. And the latter

will be found to contain sufficient information for such as want to find the true content of any single field, or parcel of land, by the chain and cross-staff only. To form the *complete* gauger and surveyer, recourse must be had to treatises written expressly for the purpose.

The practical questions, which preceeded the appendix, are now placed at the end of the book and in the same order in which they were formerly arranged.

The rest of the practical examples throughout the work, which amount to about four hundred, are given by the editor; several of which were copied from a manuscript, containing notes and observations on HAWNEY, by Mr. RAWES, master of the academy at *Bromley*, in *Kent*.

In this edition, *eighty* new geometrical figures, illustrating the different problems, are likewise given; so that neither pains nor expence have been spared, to render this work of equal utility with any other of a similar nature.

Heddon-Court, Swallow-street,
September, 1798.

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COMPLETE MEASURER.

PART I.

CHAPTER I.

Notation of Decimals.

A DECIMAL fraction is an artificial way of setting down and expressing natural, or vulgar fractions, as whole numbers. A decimal fraction has always for its denominator an unit, with a cypher or cyphers annexed to it, and must therefore be either 10, 100, 1000, 10000, &c. and consequently in writing down a decimal fraction there is no necessity for writing down the denominator: as by bare inspection, it is certainly known, consisting of an unit with as many cyphers annexed to it as there are places (or figures) in the numerator.

EXAMPLES. The decimal fraction $\frac{25}{100}$ may be written thus, .25, its denominator being known to be an unit with two cyphers; because there are two figures in the numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125: $\frac{3575}{10000}$ thus, .3575; $\frac{75}{1000}$ thus, .075; and $\frac{65}{10000}$ thus, .0065.

As whole numbers increase in a ten-fold proportion, towards the left hand, so on the contrary, decimals decrease towards the right hand in the same proportion. as in the following table.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thousandth Parts.	Ten Thousandth Parts.	Hundred Thousandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0,	1	2	3	4	5	6

Hence it appears, that cyphers put on the right hand of whole numbers, increase the value of those numbers in a ten-fold proportion: But being annexed to the right hand of a decimal fraction, neither increase nor decrease the value of it: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, though in whole numbers, cyphers before them, neither increase nor diminish the value; yet cyphers before a decimal fraction diminish its value in a ten-fold proportion: For .25, if you put a cypher before it, becomes $\frac{025}{1000}$ or .025: And .125 is $\frac{00125}{100000}$, by prefixing two cyphers thus, .00125. And therefore when you are to write a decimal fraction, whose denominator has more cyphers than there are figures in the numerator, the places of such figures must be supplied by placing cyphers before the figures of your numerator; as, suppose $\frac{19}{1000}$ were to be written down, without its denominator; here, because there are three cyphers in the denominator, and but two figures in the numerator, therefore put a cypher before 19, and set it down thus, .019.

CHAPTER II.

Reduction of Decimals.

IN REDUCTION OF DECIMALS, there are three cases: 1st, To reduce a vulgar fraction to a decimal. 2dly, To find the value of a decimal in the known parts of coin, weights, measures, &c. and 3dly, To reduce coin, weights, measures, &c. to a decimal.

I. *To reduce a Vulgar Fraction to a Decimal.*

THE RULE.

As the denominator of the given fraction is to its numerator, so is an unit (with a competent number of cyphers annexed) to the decimal required.

Therefore, if to the numerator given, you annex a competent number of cyphers, and divide the result by the denominator, the quotient is the decimal equivalent to the vulgar fraction given.

EXAMPLE 1. Let $\frac{3}{4}$ be given, to be reduced to a decimal of two places, or having 100 for its denominator.

To 3 (the numerator given) annex two cyphers, and it makes 300, which divide by the denominator 4, and the quotient is .75, the decimal required, and is equivalent to $\frac{3}{4}$ given.

NOTE. That so many cyphers as you annex to the given numerator, so many places must be pointed off in the decimal found; and if it should happen, that there are not so many places of figures in the quotient, the deficiency must be supplied, by prefixing cyphers to the quotient figures, as in the next example.

EXAMPLE 2. Let $\frac{3}{573}$ be reduced to a decimal having six places.

To the numerator annex six cyphers, and divide by the denominator, and the quotient is 5235, but it was required to have six places, therefore you must put two cyphers before it, and then it will be .005235, which is the decimal required, and is equivalent to $\frac{3}{573}$.

See the work of these two examples.

```
4)3.00(.75          573)3.000000(.005235
  28                      . . . .
  ——                      ————
   20                     1350
   20                     ————
  ——                      2040
   . .                    ————
    .                     3210
                          ————
                           345
```

In the second example there remains 345, which remainder is very insignificant, it being less than $\frac{1}{1000000}$ part of an unit, and therefore is rejected.

PRACTICAL EXAMPLES.

3. Reduce $\frac{4}{7}$ to a decimal. *Ans.* .57142 *rem.* 4.
4. Reduce $\frac{1}{243}$ to a decimal.
Ans. .0041152263 *rem.* 91.
5. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{8}$ to a decimal.
Ans. .20833, &c.
6. Reduce 15 $\frac{5}{13}$ to a mixed decimal.
Ans. 15.38461 *rem.* 7.
7. Reduce $\frac{5}{29}$ to a decimal.
Ans. .17241379 *rem.* 9.
8. Reduce $\frac{5}{191}$ to a decimal.
Ans. .026178010471 *rem.* 39.

Note. A finite decimal is that which ends at a certain number of places, such for instance as example 1. But an infinite decimal is that which no where ends, but is understood to be indefinitely continued, such as example 3.—In short, all fractions whatever, whose denominators are not composed of 2 or 5, or both, will have their correspondent decimal infinite. The method of managing circulating decimals may be met with in *Keith's* Arithmetic, and several others; but for common use, all the decimals, beyond three or four places, may be safely rejected, without affecting the truth of the conclusion.

II. *To find the value of a Decimal in the known parts of money, weight, measure, &c.*

THE RULE.

Multiply the given decimal by the number of parts in the next inferior denomination, and from the product point off so many figures to the right hand as there were figures in the decimal given; and multiply those figures pointed off by the number of parts in the next inferior denomination, and point off so many places as before, and thus continue to do till you have brought it to the lowest denomination required.

Example. 1. Let .7565 of a pound sterling be given to be reduced to shillings, pence and farthings.

Multiply by 20, by 12, and by 4, as the rule directs, and always point off four figures to the right hand, and you will find it make 15*s.* 1*d.* 2*q.* See the work.

$$\begin{array}{lr} & .7565 \\ & 20 \\ \hline s. & 15.1300 \\ & 12 \\ \hline d. & 1.5600 \\ & 4 \\ \hline q. & 2.2400 \end{array}$$

EXAMPLE 2. Let .59755 of a pound troy be reduced to ounces, penny-weights and grains.

Multiply by 12, by 20, and by 24, and always point off five figures towards the right hand, and you will find the answer to be 7 *oz.* 3 *dwts.* 10. *gr.* *fere.* See the work.

```
 .59755
     12
 -------
7.17060
     20        oz. pwts.  gr.
 -------  Facit 7    3   9.888
3.41200
     24
 -------
 164800
 82400
 -------
9.88800
```

EXAMPLE 3. Let .43569 of a ton be reduced to hundreds, quarters, and pounds.

Multiply by 20, by 4, and by 28, and the answer will be 8 *C.* 2 *qrs.* 24 *lb.* *fere.*

```
  .43569
      20
 --------
 8.71380
       4          C.  qrs.  lb.
 --------  Facit  8    2   23.9456
 2.85520
      28
 --------
23.94560
```

EXAMPLE. 4. Let .9595 of a foot be reduced into inches and quarters.

```
 .9595
    12
 -------
11.5140
     4     Facit 11 inches 2 quarters.
 -------
 2.0560
```

PRACTICAL EXAMPLES.

5. What is the value of .7575 of a pound sterling?
Ans. 15s. $1\frac{3}{4}$d. .2.

6. Required the value of .75435 of a shilling?
Ans. 9.0522 pence.

7. What is the value of .375 of a guinea?
Ans. 7s. $10\frac{1}{2}$d.

8. What is the value of .4575 of a hundred weight?
Ans. 1 qr. 23 lb. 3 oz. 13.44 drams.

9. What is the value of .175 of a ton avoirdupois?
Ans. 3 cwt. 2 qrs.

10. Required the value of .02575 of a pound troy?
Ans. 6 dwt. 4.32 grs.

11. What is the value of .04535 of a mile?
Ans. 14 p. 2 yds. 2 ft. 5 in. 1.128 barley-corn.

12. What is the value of .6375 of an acre?
Ans. 2 roods 22 perches.

13. What is the value of .574 of a hogshead of beer? *Ans.* 30 gal. 3 qt. 1.968 pt.

14. What is the value of .4285 of a year?
Ans. 156 days, 12 hrs. 13 m. 51 sec. 36 thirds.

III. *To reduce the known parts of money, weights, measure, &c. to a decimal.*

THE RULE.

To the number of parts of the less denomination given, annex a competent number of cyphers, and

divide by the number of such parts that are contained in the greater denomination, to which the decimal is to be brought; and the quotient is the decimal sought.

Example 1. Let 6*d.* be reduced to the decimal of a pound.

To 6 annex a competent number of cyphers (suppose 3,) and divide the result by 240 (the pence in a pound,) and the quotient is the decimal required.

```
240)6.000(.025
    ------
     1200
     ----   Facit .025
     ....
```

Example 2. Let 3*d.* ¾ be reduced to the decimal of a pound, having six places.

In 3*d.* ¾ there are fifteen farthings, therefore to 15 annex six cyphers (because there are to be six places in the decimal required,) and divide by 960 (the farthings in a pound,) and the quotient is .015625.

```
96|0)15.00000|0(.015625
       ....
      -----
       540
       ---
       600
       ---
       240
       ---
        480
        ---
         ..
```

Example. 3. Let 3 ¼ inches be reduced to the decimal of a foot, consisting of four places.

In $3\frac{1}{4}$ inches, there are 13 quarters; therefore to 13 annex four cyphers, and divide by 48 (the quarters in a foot) and the quotient is .2708.

```
48)13.0000(.2708
   ------
     340
     ---
     400
     ---
      16
```

Example 4. Let 9 *C.* 1 *qr.* 16 *lb.* be reduced to the decimal of a ton, having 6 places.

```
C.  qr.  lb.
9   1    16          2240)1052.000000(.469642
4                               ......
--                         -----------
37 qrs.                        15600
28                             -----
---                            21600
302                            -----
75                             14400
---                            -----
1052 Pounds                     9600
                               -----
                                6400
                               -----
                                1920
```

Facit .469642.

PRACTICAL EXAMPLES.

5. Reduce 7s. $5\frac{1}{2}$d. to the decimal of a pound?
Ans. *l*.3729166, &c.

6. What decimal part of a pound sterling is three-halfpence? *Ans.* *l*.00625.

7. Reduce 4s. $7\frac{9}{11}$d. to the decimal of a pound sterling? *Ans.* *l*.2325757, &c.

8. Reduce 10 *oz.* 11 *dwt.* 3 *gr.* to the decimal of a pound Troy? *Ans.* .1296875 *lb.*

9. Reduce 3 *cwt.* 1 *qr.* 14 *lb.* to the decimal of a Ton? *Ans.* .16875 Ton.

10. Reduce 22 feet 7 inches to the decimal of a Foot? *Ans.* 22.5833 Feet.

11. Reduce 2 *qrs.* 15 *lb.* to the decimal of a hundred weight? *Ans.* .6336285714, &c. *cwt.*

12. What decimal part of a year is 3*w.* 4*d.* 55 hours, reckoning 365 days 6 hours a Year?

Ans. .074720511065 *yr.*

13. Reduce 2.45 shillings to the decimal of a pound? *Ans.* *l* .1225.

14. Reduce 1.074 roods to the decimal of an Acre? *Ans.* .2685 Acre.

15. Reduce 17.69 yards to the decimal of a mile? *Ans.* .010051136 *m.*

CHAPTER III.

Addition of Decimals.

ADDITION of decimals is performed the same way as Addition of whole numbers, only you must observe to place your numbers right, that is all the decimal points under each other, units under units, tenths under tenths, &c.

Example. Let 317.25; 17.125; 275.5; 47.3579; and 12.75; be added together into one sum.

```
    317.25
     17.125
    275.5
     47.3579
     12.75
    ---------
Sum 669.9829
```

PRACTICAL EXAMPLES.

2. Add 5.714; 3.456; .543; 17.4957 together.
Sum 27.2087.
3. Add 3.754; 47.5; .00857; and 37.5 together.
Sum 88.76257.
4. Add 54.34; .375; 14.795; and 1.5 together.
Sum 71.01.

CHAPTER IV.

Subtraction of Decimals.

SUBTRACTION of decimals is likewise performed the same way as in whole numbers, respect being had (as in addition) to the right placing of the numbers, as in the following examples.

	(1)		(2)
From	212.0137	From	201.125
Subtr.	31.1275	Subtr.	5.57846
Rests	180.8862	Rests	195.54654

Note. If the number of places in the decimals be more in that which is to be subtracted, than in that which you subtract from, you must suppose cyphers to make up the number of places as in the second Example.

PRACTICAL EXAMPLES.

3. Required the difference between 57.49 and 5.768?
Ans. 51.722.

4. What is the difference between .3054 and 3.054? *Ans.* 2.7486.

5. Required the difference between 1745.3 and 173.45? *Ans.* 1571.85.

6. What is the difference between seven tenths of an unit, and fifty-four ten thousand parts of an unit? *Ans.* .6946.

7. What is the difference between .105 and 1.00075? *Ans.* .89575.

8. What is the difference between 150.43 and 755.355? *Ans.* 604.925.

9. From 1754.754 take 375.494478? *Ans.* 1379.259522.

10. Required the difference between 17.541 and 35.49? *Ans.* 17.949.

CHAPTER V.

Multiplication of Decimals.

MULTIPLICATION of decimals is also performed the same way as Multiplication of whole numbers; but to know the value of the product, observe this Rule:

Cut off, or separate by a comma or point, so many decimal places in the product, as there are places of decimals in both factors, viz. both in the multiplicand and multiplier.

EXAMPLE 1. Let 3.125 be multiplied by 2.75.

Multiply the numbers together, as if they were whole numbers, and the product is 8.59375: And because there are three places of decimals pointed off in the multiplicand, and two places in the multiplier, therefore you must point off five places of decimals in the product, as you may see by the work.

Multiplicand	3.125
Multiplier	2.75
	15625
	21875
	6250
Product	8.59375

EXAMPLE 2. Let 79.25 be multiplied by .459.

In this example, because two places of decimals are pointed off in the multiplicand, and three in the multiplier, therefore there must be five pointed off in the product.

Multiplicand	79.25
Multiplier	.459
	71325
	39625
	31700
Product	36.37575

EXAMPLE 3. Let 1.35272 be multiplied by .00425.

In this example, because in the multiplicand are six decimal places and in the multiplier five places; therefore in the product there must be eleven places of decimals; but when the multiplication is finished, the product is but 57490600 viz. only eight places; therefore, in this case, you must put three cyphers before the product figures, to make up the number of eleven places: So the true product will be .00057490600.

Multiplicand	.135272
Multiplier	.00425
	676360
	270544
	541088
Product	.00057490600

PRACTICAL EXAMPLES.

4. Multiply .001472 by .1045.
Product .0001538240.
5. Multiply .017532 by 347. *Product* 6.083604.
6. Multiply 279.25 by .445. *Product* 124.26625.
7. Multiply 32.0752 by .0325.
Product 1.04244400.
8. Multiply 4.443 by 15.98. *Product* 70.99914.
9. Multiply 20.0291 by 35.45.
Product 710.031595.
10. Multiply 7.3564 by .0126.
Product .09269064.
11. Multiply .75432 by .0356.
Product .026853792.
12. Multiply .004735 by .0374.
Product .0001770890.

Contracted MULTIPLICATION *of* DECIMALS.

Because in multiplication of decimal parts, and mixed numbers, there is no need to express all the figures of the product, but in most cases two, three or four places of decimals will be sufficient; therefore, to contract the work, observe the following

RULE.

Write the unit's place of the multiplier under that place of the multiplicand, which you intend to keep in the product: then invert the order of all the other figures; that is write them all the contrary way; and in multiplying, begin always at that figure in the multiplicand which stands over the figure you are then multiplying withal, and set down the first figure of each particular product directly one under the other: But take care to increase the first figure of every line

of the product, with what would arise by carrying 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35, &c. from the product of the two figures (in the multiplicand) on the right-hand of the multiplying figure.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four places retained in the decimals of the product.

First, according to the directions, write down the multiplicand, and under it write the multiplier, thus: place the 8 (being the unit's place of the multiplier) under 4, the fourth place of decimals in the multiplicand, and write the rest of the figures quite contrary to the usual way, as in the following work: Then begin to multiply, first the 5 which is left out, only with regard to the increase, which must be carried from it; saying, 8 times 5 is 40; carry four in your mind, and say, 8 times 4 is 32, and 4 I carry, is 36; set down 6, and carry 3, and proceed through the rest of the figures as in common multiplication. Then begin to multiply with 2; saying, 2 times 5 is 10, nought and carry 1; 2 times 4 is 8, and 1 is 9, for which I carry 1, because it is above 5; then 2 times 6 is 12, and 1 that I carry is 13; set down 3 and carry 1; and proceed through the rest of the figures as in common multiplication. Then multiply with 1: saying, once 6 is 6, for which I carry 1, and say, once 8 is 8, and 1 is 9; set down 9, and proceed as in common multiplication. Then multiply with 7: saying, 7 times 6 is 42, 2 and carry 4; 7 times 8 is 56, and 4 is 60, nought and carry 6; 7 times 3 is 21, and 6 is 27; set down 7 and carry 2, and proceed. Lastly, multiply with 5: saying, 5 times 8 is 40, nought and carry 4; 5 times 3 is 15, and 4 is 19; for which carry 2, and say, 5 times 2 is 10, and 2 that I carry is 12; which set down, and add all the products together, and the total will be 19.6107.—See the work.

Contracted.	*Common.*	
2.38645	2.38645	
5712.8	8.2175	
190916	11	93225
4773	167	0515
239	238	645
167	4772	90
12	190916	0
19.6107	19.6106	52875

I have here set down the work of the last example, wrought by the common way, by which you may see the reason of the contracted way, all the figures on the right-hand of the line being wholly omitted.

Example 2. Let 375.13758 be multiplied by 16.7324, so that the product may have but four places of decimals.

375.13758 the Multiplicand.
4237.61 the Multiplier reversed.

37513758 the product with 1
22508255 the prod. with 6 increased with 6×8
2625963 the prod. with 7 increased with $7 \times 8 \times 5$
112541 the prod. with 8 increased with $3 \times 5 \times 7$
7503 the prod. with 2 increased with $2 \times 7 \times 3$
1500 the prod. with 4 increased with 0

6276.9520 the product required.

PRACTICAL EXAMPLES.

Example 3. Multiply 395.3756 by .75642.

Ans. 299.0699.

4. Let 54.7494367 be multiplied by 4.724753 reserving only five places of decimals in the product.

Ans. 258.67756.

5. Multiply 475.710564 by .3416494 and retain three decimals in the product. *Ans.* 162.525.

6. Let 4745.679 be multiplied by 751.4549, and reserve only the integers, or whole numbers, in the product. *Ans.* 3566163.

CHAPTER VI.

Division of Decimals.

DIVISION of decimals is performed in the same manner as division of whole numbers: to know the value or denomination of the quotient, is the only difficulty; for the resolving of which, observe either of the following

RULES.

I. The first figure in the quotient must be of the same denomination with that figure in the dividend which stands (or is supposed to stand) over the unit's place of the product of the first quotient figure by the divisor.

II. When the work of division is ended, count how many places of decimal parts there are in the dividend more than in the divisor; for that excess is the number of places which must be separated in the quotient for decimals. But if there be not so many figures in the quotient as there are in the said excess, that deficiency must be supplied, by placing cyphers before the significant figures, towards the left-hand, with a point before them; and thus you will plainly discover the value of the quotient.

These following directions ought also to be carefully observed.

If the divisor consist of more places than the dividend, there must be a competent number of cyphers annexed to the dividend, to make it consist of as many (at least) or more places of decimals than the divisor; for the cyphers added must be reckoned as decimals.

Consider whether there be as many decimal parts in the dividend as there are in the divisor; if there be not, make them so many, or more, by annexing cyphers.

In dividing whole or mixed numbers, if there be a remainder, you may bring down more cyphers; and, by continuing your division, carry the quotient to as many places of decimals as you please.

Example 1. Let 48 be divided by 144.

In this example the divisor 144 is greater than the dividend 48; therefore, according to the directions above, I annex a competent number of cyphers (*viz.* four,) with a point before them, and divide in the usual way.

```
144)48.0000(.3333
    432
    ———
     480
     ———
      480
      ———
       480
       ———
        48
```

But, first, in seeking how often 144 in 48.0 (the first three figures of the dividend,) I find the unit's place of the product of the first quotient figure by the

divisor to fall under the first place of decimals; therefore the first figure in the quotient is in the first place of decimals: Or, by the second rule, there being four places of decimals in the dividend, and none in the divisor; so the excess of decimal places in the dividend, above that in the divisor, is four; so that when the division is ended, there must be four places of decimals in the quotient.

EXAMPLE 2. Let 217.75 be divided by 65.

First, in seeking how often 65 in 217 (the first three figures of the dividend) I find the unit's place of the product of the first quotient figure by the divisor to fall under the unit's place of the dividend; therefore the first figure in the quotient will be units, and all the rest decimals: Or, by the second rule, there being two places of decimals in the dividend, and no decimals in the divisor, therefore the excess of decimal places in the dividend, above the divisor, is two; so when the division is ended, separate two places in the quotient, towards the right-hand by a point.

```
65)217.75(3.35
   195 . .
   -------
    227
   -------
     325
   -------
     . . .
```

EXAMPLE 3. Let 276.15975 be divided by 13.25.

```
13.25)267.15975(20.163
      2650 . . . .
      ---------
       2159
         8347
          3975
      ---------
```

In this third example the unit's place of the product of the first quotient figure by the divisor falls under 6, the ten's place of the dividend; therefore, (by the first rule) the first figure in the quotient is tens: Or, by the second rule, the excess of decimal places in the dividend, above the divisor, is three; there being five places of decimals in the dividend, and but two in the divisor, so there must be three places of decimals in the quotient.

Example 4. Let 15.675159 be divided by 375.89.

```
375.89)15.675159(.0417
        150356
        ------
         63955
        ------
        263669
        ------
           546
```

In this fourth example, the unit's place of the product of the first quotient figure by the divisor, falls under 7, the second place of decimals in the dividend; therefore (by the first rule) the first figure in the quotient is in the second place of decimals; so that you must put a cypher before the first figure in the quotient; and by the second rule, the excess of decimal places in the divisor is 4; for the decimal places in the dividend are 6, and the number of places in the divisor but two; therefore there must be four places of decimals in the quotient: But the division being finished after the common way, the figures in the quotient are but three, therefore you must put the cypher before the significant figures.

EXAMPLE 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
       6735 . .
       ------
        4806
        -----
        7654
        -----
         9190
         -----
         11080
         -----
           304
```

In this example, the divisor being a decimal, the last figure of the product of the first quotient figure by the divisor falls under the ten's place in the dividend, therefore the units (if there had been any) would fall under the hundreds place in the dividend, and so the first figure in the quotient is hundreds. And by the second rule, there being four places of decimals in the dividend, as many in the divisor, so the excess is nothing; but in dividing I put two cyphers to the remainders, and continue the division to two places further; so I have two places of decimals.

EXAMPLE 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
        0914 . . .
        -----
         3360
         -----
          1610
          -----
          2390
          -----
           .105
```

In this example, the unit's place of the product of the first quotient figure by the divisor (if there had been any) would fall under the unit's place of the dividend: therefore the first figure of the quotient is units. And, by the second rule, their being seven places of decimals in the dividend, and but four places in the divisor, so the excess is three; therefore there must be three places of decimals in the quotient.

PRACTICAL EXAMPLES.

7. Divide .0000059791 by .00456. *Quotient* .00131
8. Divide an unit by 282, or, in other words, find the reciprocal of 282. *Quotient* .0035461
9. Divide .4 by .325. *Quotient* 1.2307
10. Divide 495 by .042. *Quotient* 11785.71
11. Divide .475321 by 97.453. *Quotient* 0048774
12. Divide 17.543275 by 125.7. *Quotient* .13956
13. Divide 143754.35 by .7493. *Quotient* 191851.528
14. Divide 16 by 960. *Quotient* .01666, &c.
15. Divide 12 by 1728 *Quotient* .006944, &c.
16. Divide 47.5493 by 34.75 *Quotient* 1.36832517
17. Divide 70.3571 by .00573. *Quotient* 12976.3062
18. Divide .3754 by 75.714. *Quotient* .004958131

Division of Decimals contracted.

In division of decimals the common way, when the divisor has many figures, and it is required to continue the division till the value of the remainder be but small, the operation will sometimes be long and tedious, but may be contracted by the following method.

THE RULE.

By the first rule of this chapter (page 17,) find what is the value of the first figure in the quotient: then by knowing the first figure's denomination, you may have as many or as few places of decimals as you please, by taking as many of the left hand figures of the divisor as you think convenient for the first divisor; and then take as many figures of the dividend as will answer them; and, in dividing, omit one figure of the divisor at each following operation; observing to carry for the increase of the figures omitted, as in contracted multiplication.

Note. When there are not so many figures in the divisor as are required to be in the quotient, begin the division with all the figures, as usual, and continue dividing till the number of figures in the divisor is equal to the number of figures remaining to be found in the quotient; after which use the contraction.

Example 1. Let 721.17562 be divided by 2.257432; & let there be three places of decimals in the quotient.

Contracted.	*Common.*	
2.25743)721.175—62(319.467	2.25743)721.17562(319.467	
.....677229	677229	..
43946	43946	6
22574	22574	3
21372	21372	32
20317	20316	87
1055	1055	450
903	902	972
152	152	4780
135	135	4458
17	17	03220
16	15	80201
	1	23019

In this example, the unit's place of the product of the first quotient figure by the divisor falls under the hundred's place in the dividend, and it is required, that three places of decimals be in the quotient, so there must be six places in all; that is, three places of whole numbers, and three places of decimals. Then, because I can have the divisor in the first six figures of the dividend, I cut off the 62 with a dash of the pen, as useless; then I seek how often the divisor is in the dividend, and the answer is three times; put three in the quotient, and multiply and subtract as in common division, and the remainder is 43946. Then point off three in the divisor, and seek how often the remaining figures may be had in 43946, the remainder, which can be but once; put 1 in the quotient, and multiply and subtract, and the next remainder is 21372. Then point off the 4 in the divisor, and seek how often the remaining figures may be had in 21372, which will be 9 times; put 9 in the quotient; multiply as in contracted multiplication, and thus proceed till the division is finished.

I have set down the work of this example at large, according to the common way, that thereby the learner may see the reason of the rule; all the figures on the right-hand side of the perpendicular line being wholly omitted.

PRACTICAL EXAMPLES.

2. Let 5171.59165 be divided by 8.758615, and let it be required, that four places of decimals be pointed off in the quotient. *Ans.* 590.4577

3. Let 25.1367 be divided by 217.3543, and let there be five places of decimals in the quotient. *Ans.* .11564.

4. Divide 7414.76717 by 2.756756, and let there be five places of decimals in the quotient. *Ans.* 2689.67118

5. Divide 514.75498 by 12.34254, and let there be six places of decimals in the quotient.

Ans. 41.705757.

6. Divide 47194.379457 by 14.73495. and let the quotient contain as many decimal places as there will be integers, or whole numbers, in it.

Ans. 3202.8869.

CAA TER VII.

Extraction of the Square Root.

If a square number be given;

TO find the Root thereof, that is, to find out such a number, as being multiplied into itself, the product shall be equal to the number given; such operation is called, *The Extraction of the Square Root;* which to do, observe the following directions.

1st, You must point your given number; that is, make a point over the unit's place, another over the hundred's, and so over every second figure throughout.

2dly, Then seek the greatest square number in the first period towards the left hand, placing the square number under that point, and the root thereof in the quotient, and subtract the said square number from the first point, and to the remainder bring down the next point, and call that the resolvend.

3dly, Then double the quotient, and place it for a divisor on the left hand of the resolvend; and seek how often the divisor is contained in the resolvend (reserving always the unit's place) and put the answer in the quotient, and also on the right hand side of the divisor; then multiply by the figure last put in the quotient, and subtract the product from the resolvend (as in common division) and bring down the next point

to the remainder (if there be any more) and proceed as before.

A Table of Squares and their Roots.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

EXAMPLE 1. Let 4489 be a number given, and let the square root thereof be required.

```
   . .
   4489(67
   36
  -------
127)889    Resolvend.
    889    Product.
  -------
    ...
```

First, point the given number, as before directed; then by the little table foregoing, seek the greatest square number in 44 (the first point to the left-hand) which you will find to be 36, and 6 the root; put 36 under 44, and 6 in the quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other point 89, placing it on the right-hand, so it makes 889 for a resolvend; then double the quotient 6, and it makes 12; which place on the left-hand for a divisor, and seek how often 12 in 88 (reserving the unit's place) the answer is 7 times; which put in the quotient, and also on the right-hand side of the divisor, and multiply 127 by 7, as in common division, and the product is 889, which subtracted from the resolvend, there remains nothing; so is your work finished; and the square root of 4489 is 67; which root if you multiply by itself, that is 67 by 67, the product will be 4489, equal to the given square number, and proves the

work to be right. Had there been any remainder it must have been added to the square of the root found.

EXAMPLE 2. Let 106929 be a number given, and let the square root thereof be required.

```
    . . .
   106929(327
    9
  ----------
  62)169      Resolvend.
     124      Product.
  ----------
 647)4529     Resolvend.
     4529     Product.
     ----
     ....
```

First, point your given number, as before directed, putting a point over the units, hundreds, and tens of thousands; then seek what is the greatest square number in 10 (the first point) which by the little table you will find to be 9, and 3 the root thereof; put 9 under 10, and 3 in the quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next point, and it makes 169 for the resolvend; then double the quotient 3, and it makes 6, which place on the left-hand of the resolvend for a divisor, and seek how often 6 in 16; the answer is twice; put 2 in the quotient, and also on the right-hand of the divisor making it 62. Then multiply 62 by the 2 you put in the quotient, and the product is 124; which subtract from the resolvend, and there remains 45; to which bring down 29, the next point, and it makes 4529 for a new resolvend. Then double the quotient 32, and it makes 64, which place on the left side of the resolvend for the divisor, and seek how often 64 in 452, which you will find 7 times: put 7 in the quotient, and also on the right-hand of the divisor, making it 647, which multiplied by the 7 in the quotient, it makes

4529, which subtracted from the resolvend, there remains nothing. So 327 is the square root of the given number.

NOTE. The root will always contain just so many figures, as there are points over the given number to be extracted: And these figures will be whole numbers or decimals respectively, according as the points stand over whole numbers or decimals.—The method of extracting the square root of a decimal is exactly the same as in the foregoing examples, only if the number of decimals be odd, annex a cypher to the right hand to make them even, before you begin to point. The root may be continued to any number of figures you please, by annexing two cyphers at a time to each remainder, for a new resolvend.

PRACTICAL EXAMPLES.

3. It is required to extract the square root of 2268741. *Ans.* 1506.23. *Rem.* 121871.

4. What is the square root of 7596796?
Ans. 2756.228. *Rem.* 3212016.

5. What is the square root of 751427.5745?
Ans. 866.84. *Rem.* 59889.

6. Extract the square root of 656714.37512.
Ans. 810.379. *Rem.* 251479.

7. What is the square root of 15241578750190521?
Ans. 123456789.

8. What is the square root of 75.347?
Ans. 8.6802649729. *Rem.* 24536226559.

9. What is the square root of .4325?
Ans. .65764. *Rem.* 96304.

To extract the Square Root of a Vulgar Fraction.

RULE.

1. Reduce the given fraction to its lowest terms, if

it be not in its lowest terms already; then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator.

2. If the fraction will not extract even, reduce it to a decimal, and then extract the square root.

3. When the number to be extracted is a mixed fraction, reduce the fractional part to a decimal, and annex it to the whole number, then extract the square root.

EXAMPLE 1. Extract the square root of $\frac{175}{252}$

First, $\frac{175}{252}$ is equal to $\frac{25}{36}$ in its lowest terms, the square root of 25 is 5, and the square root of 36 is 6; therefore $\frac{5}{6}$ is the root required.

EXAMPLE 2. Let seven-eighths be a vulgar fraction given, whose square root is required.

8)7.000	(.87500000(.9354
64	81
60	183)650
56	549
40	1865)10100
40	9325
	18704)77500
	74816
	2684

Reduce this $\frac{7}{8}$ to a decimal, it makes .875; to which annex cyphers, and extract the square root, as if it was a whole number. So the root is .9354.

EXAMPLE 3. Let $\frac{3}{960}$ be a vulgar fraction, whose square root is required.

96l0)3.000000	(.00312500(.0559 Root.
288	25
120	105)625
96	525
240	1109)10000
192	9981
480	19
480	
. . .	

EXAMPLE 4. What is the square root of $15\frac{5}{8}$?
Here $\frac{5}{8}$ reduced to a decimal is .625, which annexed to the 15 makes 15.625, the square root of which is 3.95284. *Rem.* 559344.

PRACTICAL EXAMPLES.

5. What is the square root of $\frac{448}{567}$? *Ans.* $\frac{5}{9}$.
6. What is the square root of $\frac{275}{539}$? *Ans.* $\frac{5}{7}$.
7. What is the square root of $\frac{4\frac{1}{2}}{9\frac{1}{4}}$?
Ans. .6918984, &c.
8. What is the square root of $29\frac{4}{25}$? *Ans.* 5.4.
9. What is the square root of $\frac{1}{8}$?
Ans. 3535. *Rem.* 3775.

CHAPTER VIII.

Extraction of the Cube Root.

TO extract the cube root, is nothing else but to find such a number, as being first multiplied into itself, and then into that product, produceth the given number; which to perform, observe the following directions.

1st, You must point your given number, beginning with the unit's place, and make a point, or dot, over every third figure towards the left-hand.

2dly, Seek the greatest cube number in the first point, towards the left-hand, putting the root thereof in the quotient, and the said cube number under the first point, and subtract it therefrom, and to the remainder bring down the next point, and call that the resolvend.

3dly, Triple the quotient, and place it under the resolvend; the unit's place of this under the ten's place of the resolvend; and call this the triple quotient.

4thly, Square the quotient, and triple the square, and place it under the triple quotient; the units of this under the ten's place of the triple quotient, and call this the triple square.

5thly, Add these two together, in the same order as they stand, and the sum shall be the divisor.

6thly, Seek how often the divisor is contained in the resolvend, rejecting the unit's place of the resolvend (as in the square root,) and put the answer in the quotient.

7thly, Cube the figure last put in the quotient, and put the unit's place thereof under the unit's place of the resolvend.

8thly, Multiply the square of the figure last put in the quotient into the triple quotient, and place the product under the last, one place more to the left-hand.

9thly, Multiply the triple square by the figure last put in the quotient, and place it under the last, one place more to the left-hand.

10thly, Add the three last numbers together, in the same order as they stand, and call that the subtrahend.

Lastly, Subtract the subtrahend from the resolvend, and if there be another point, bring it down to the remainder, and call that a new resolvend, and proceed in all respects as before.

NOTE. To square a number is to multiply that number by itself. And,
To cube a number is to multiply the square of the number by the number itself.

A Table of Cubes and their Roots.

Roots 1	2	3	4	5	6	7	8	9
Cubes 1	8	27	64	125	216	343	512	729

Example 1. Let 314432 be a cubic number, whose root is required.

```
 . .
314432(68 Root.
216
—————
98432 Resolvend.
—————
  18 Triple quotient of 6.
 108  Triple square of the quotient 6.
—————
1098 Divisor.
—————
  512 Cube of 8, the last figure of the root.
 1152  The square of 8, by the triple quotient.
 864   The triple square of the quotient 6 by 8.
—————
98432 The subtrahend.
—————
.....
```

After you have pointed the given number, seek what is the greatest cube number in 314, the first point, which, by the little table annexed to the rule you will find to be 216, which is the nearest that is less than 314, and its root is 6; which put in the quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next point, 432, and annex it to 98; so will it make 98432 for the resolvend. Then triple the quotient 6, it makes 18, which write down the unit's place, 8, under 3, the ten's place of the resolvend. Then square the quotient 6, and triple the square, and it makes 108, which write under the triple quotient, one place towards the left-hand; then add those two numbers together, and they make 1098 for the divisor. Then seek how often the divisor is contained in the resolvend, (rejecting the unit's place thereof) that is, how often 1098 in 9843,

which is 8 times; put 8 in the quotient, and the cube thereof below the divisor, the unit's place under the unit's place of the resolvend. Then square the 8 last put in the quotient, and multiply 64, the square thereof, by the triple quotient 18; the product is 1152; set this under the cube of 8, the units of this under the tens of that. Then multiply the triple square of the quotient by 8, the figure last put up in the quotient, the product is 864; set this down under the last product, a place more to the left-hand. Then draw a line under these three, and add them together, and the sum is 98432, which is called the subtrahend; and being subtracted from the resolvend, the remainder is nothing; which shews the number to be a true cubic number, whose root is 68; that is, if 68 be cubed, it will make 314432.

For if 68 be multiplied by 68, the product will be 4624; and this product, multiplied again by 68, the last product is 314432, which shews the work to be right.

Example 2. Let the cube root of 5735339 be required.

After you have pointed the given number, seek what is the greatest cube number in 5, the first point, which, by the little table, you will find to be 1; which place under 5, and 1, the root thereof, in the quotient; and subtract 1 from 5, and there remains 4; to which bring down the next point, it makes 4735 for the resolvend. Then triple the 1, and it makes 3; and the square of 1 is 1, and the triple thereof is 3; which set one under another, in their order, and added, makes 33 for the divisor. Seek how often the divisor goes in the resolvend, and proceed as in the last example.

5735339(179 Root.
1

4735

3 The triple of the quotient 1, the first figure.
3 The triple square of the quotient 1.

33 The divisor.

343 The cube of 7, the second figure of the root.
147 The square of 7, multipl. in the triple quot. 3.
21 The triple square of the quot. multiplied by 7.

3913 The subtrahend.

822339 The new resolvend.

51 The triple of the quot. 17, the two first fig.
867 The triple square of the quotient 17.

8721 Divisor.

729 The cube of 9, the last figure of the root.
4131 The squ. of 9, multipl. by the triple quo. 51.
7803 The triple square of the quotient 867 by 9.

822339 The subtrahend.

. . . .

In this example, 33, the first divisor, seems to be contained more than seven times in 473, the resolvend, after the unit's place has been rejected; but if you work with 9, or 8, you will find that the subtrahend will be greater than the resolvend.

EXAMPLE 3. Required the cube root of 22069810125.

```
 .  .  .  .
22069810125(2805
 8
-------
14069 Resolvend.
-------
    6 Triple of 2.
   12  Triple square of 2.
-------
  126 Divisor.
-------
  512 Cube of 8.
  384 Square of 8 by 6.
   96 Triple square by 8.
-------
13952 Subtrahend.
-------
  117810125 New resolvend.
-------------
       84 Triple of 28.
    2352  Triple square of 28.
-------
   23604 Divisor.
-------
    840 Triple of 280.
 235200 Triple square of 280.
-------
2352840 New divisor.
-------
       125 Cube of five.
     21000  Square of 5 by 840.
   1176000  Triple square by 5.
-------------
 117810125 Subtrahend.
-------------
 . . . . . . . .
```

In this example 13952, being subtracted from the resolvend 14069, the remainder is 117; to which bring down 810, the 3d. point, and it makes 117810 for a new resolvend; and the next divisor is 23604, which you cannot have in the said resolvend (the unit's place being rejected;) so you must put 0 in the quotient, and seek a new divisor (after you have brought down your last point to the resolvend;) which new divisor is 2352840; and you will find it to be contained 5 times. So proceed to finish the rest of the work.

NOTE. The root will always contain just so many figures, as there are points over the given number to be extracted; and these figures will be whole numbers or decimals respectively, according as the points stand over whole numbers or decimals. The method of extracting the cube root of a mixed number, or decimal, is the same as in the above examples; only the number of decimals must be made to consist of three, six or nine, &c. figures, by annexing cyphers.

PRACTICAL EXAMPLES.

EXAMPLE 4. What is the cube root of 32461759? *Ans.* 319

5. What is the cube root of 84604519? *Ans.* 439

6. What is the cube root of 259697989? *Ans.* 638

7. What is the cube root of 25917056? *Ans.* 295.9. *Rem.* 8995921

8. What is the cube root of 93759.57507? *Ans.* 45.42. *Rem.* 59186982

9. Required the cube root of .401719179. *Ans.* .737. *Rem.* 1403626

10. Required the cube root of .0001416? *Ans.* .052. *Rem.* 992

11. Required the cube root of 122615327232. *Ans.* 4968

12. What is the cube root of 705.919947284? *Ans.* 8.904. *Rem.* 20.

13. What is the cube root of 1551328.215978515625? *Ans.* 115.7625

14. The cube root of .57345 is required. *Ans.* .8308. *Rem.* 8045888

To extract the Cube Root of a Vulgar Fraction.

RULE.

1. Reduce the given fraction to its lowest terms, if it be not in its lowest terms already; then extract

the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator.

2. If the fraction will not extract even, reduce it to a decimal, and then extract the cube root.

3. When the number to be extracted is a mixed fraction, reduce the fractional part to a decimal, and annex this decimal to the whole number, then extract the cube root.

EXAMPLE 1. What is the cube root of $\frac{243}{576}$?

First. $\frac{243}{576}$ is equal to $\frac{27}{64}$ in its lowest terms, the cube root of 27 is 3, and the cube root of 64 is 4; therefore the cube root of $\frac{27}{64}$ is $\frac{3}{4}$, the answer.

EXAMPLE 2. Let $\frac{5}{276}$ be a vulgar fraction, whose cube root is required.

By the first rule of Chapter II. reduce the vulgar fraction to a decimal.

```
276)5.000000000(.018115942
     276 . . . . . .
    ---------------
     2240
     ----
      320
      ----
       440
       ----
       1640
       ----
       2600
       ----
        1160
        ----
         560
         ---
           8
```

$.\dot{0}18\dot{1}15\dot{9}42(.262$ Root.
8

10115 Resolvend.

6 Triple of 2.
12 Triple square of 2.

126 Divisor.

216 Cube of 6.
216 Square of 6 by the triple of 2.
72 Triple square by 6.

9576 Subtrahend.

539942 Resolvend.

78 Triple of 26.
2028 Triple square of 26.

20358 Divisor.

8 Cube of 2.
312 Square of 2 by 78.
4056 Triple square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the truth of the work, by cubing the root found, as was shewn in the first example; and if any thing remains, add it to the said cube, and the sum will be the given number, if the work is rightly performed.

EXAMPLE 3. What is the cube root of 56623 $\frac{13}{125}$?

Here $\frac{13}{125}$ reduced to a decimal is .104, which annexed to 56623 makes 56623.104, the cube root of which is 38.4.

PRACTICAL EXAMPLES.

4. What is the cube root of $\frac{175}{476}$? *Ans.* 71638
5. Required the cube root of $\frac{1912}{3078}$. *Ans.* 85324
6. What is the cube root of $\frac{81}{375}$? *Ans.* $\frac{3}{5}$.
7. What is the cube root of $\frac{5}{9}$? *Ans.* .82207
8. What is the cube root of $\frac{23}{24}$? *Ans.* .98591
9. What is the cube root of 5333$\frac{1}{3}$? *Ans.* .17.471
10. What is the cube root of 1032201$\frac{3}{5}$? *Ans.* 101.07

CHAPTER IX.

MULTIPLICATION of FEET, INCHES, and PARTS; or DUODECIMALS.

THE multiplication of *feet* and *inches* is generally called *duodecimals*, because every superior place is 12 times its next inferior in this scale of notation. This way of conceiving an unit to be divided, is chiefly in use among *artificers*, who generally take the linear dimensions of their work in *feet* and *inches*: It is likewise called *cross multiplication*, because the *factors* are sometimes multiplied crosswise.

RULE I.

1. Under the multiplicand write the corresponding denominations of the multiplier; that is, feet under feet, inches under inches, parts under parts, &c.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner multiply every term in the multiplicand by the inches in the multiplier, and set the result of each term one place removed to the right-hand of those in the multiplicand.

4. Work in a similar manner with the parts in the multiplier, setting the result of each term removed two places to the right-hand of those in the multiplicand. Proceed in like manner with the rest of the denominations, and their sum will be the answer required.

EXAMPLE 1. Let 7 feet 9 inches be multiplied by 3 feet 6 inches.

	F.		I.		
Multiplicand	7	..	9		
Multiplier	3	..	6		
	23	..	3		*Pts.*
	3	..	10	..	6
Product	27	..	1	..	6

First, Multiply 9 inches by 3, saying, 3 times 9 is 27 inches, which make 2 feet 3 inches; set down 3 under inches, and carry 2 to the feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the feet.

Then begin with 6 inches, saying, 6 times 9 is 54 parts, which is 4 inches and 6 parts; set down 6 parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 inches, which is 3 feet 10 inches; which set down, and add all up together, and the product is 27 feet 1 inch 6 parts.

Example 2. Let 7 feet 5 inches 9 parts be multiplied by 3 feet 5 inches 3 parts.

	F.	I.	P.		
Multiplicand	7 ..	5 ..	9		
Multiplier	3 ..	5 ..	3		
	22 ..	5 ..	3	S.	
	3 ..	1 ..	4 ..	9	T.
		1 ..	10 ..	5 ..	3
Product	25 ..	8 ..	6 ..	2 ..	3

In this example, I first begin with 3 feet, and thereby multiply 7 feet 5 inches and 9 parts : First, I say, 3 times 9 is 27 parts, that is 2 inches and 3 parts ; set down 3 under the parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 foot 5 inches; set down 5 inches, and carry 1, and say, 3 times 7 is 21, and 1, I carry is 22; set down 22 feet: Then begin with 5 inches, saying, 5 times 9 is 45, which is 45 *seconds*, which makes 3 parts and 9 seconds; set down 9 seconds a place towards the right-hand, and carry 3 parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 inches and 4 parts; set down 4 parts and carry 2, saying 5 times 7 is 35, and 2, I carry is 37, which is 3 feet 1 inch; set down 3 feet 1 inch ; and begin to multiply by 3 parts, saying, 3 times 9 is 27 *thirds*, that is, 2 *seconds* and 3 *thirds ;* set down 3 thirds, and carry 2, saying 3 times 5 is 15, and 2, I carry is 17, that is, 1 part and 5 seconds ; set down 5 seconds, and carry 1, saying 3 times 7 is 21, and 1, I carry is 22, which is 1 inch and 10 parts, which set down, and add all up, and the product is 25 feet 8 inches 6 parts 2 seconds 3 thirds.

RULE II.

When the Feet in the Multiplicand are expressed by a large number.

Multiply first by the feet in the multiplier, as before. Then, instead of multiplying by the inches and parts, &c. proceed as in the Rule of Practice, by taking such aliquot parts of the multiplicand as correspond with the inches and parts, &c. of the multiplier. Then the sum of them all will be the product required.

EXAMPLE 3. Let 75 feet 7 inches be multiplied by 9 feet 8 inches.

In.		F.	I.	
4		75 ..	7	Multiplicand.
4	$\frac{1}{3}$	9 ..	8	Multiplier.
		680 ..	3	
		25 ..	2 .. 4	
		25 ..	2 .. 4	
	Product	730 ..	7 .. 8	

Multiply by 9 feet, first, as above directed; then instead of multiplying by the 8 inches, let them be divided into aliquot parts of a foot, as 4 and 4, because 4 is the third part of 12. So, if you take the third part of 75 feet 7 inches, and set it down twice, and add all together, the sum will be 730 feet 7 inches 8 parts. To take the third part, say, how often 3 in 7, which is twice; set down 2; then because twice 3 is 6, say, 6 out of 7, and there remains 1, for which you must add 10 to the 5, and it makes 15; then the threes in 15 are 5 times: set down 5: and, because three times 5 is 15, there is 0 remains. Then go to the 7 inches, saying, the three in 7 are twice: set down 2

in the inches; and because twice 3 is 6, take 6 out of 7, and there remains 1 inch, which is 12 parts; then threes in 12 are 4 times, and 0 remains. So the third part of 75 feet 7 inches is 25 feet 2 inches 4 parts; which set twice over, and add them together as in the example.

Example 4. Let 37 feet 7 inches 5 parts be multiplied by 4 feet 8 inches 6 parts.

I. P.		F.	I.	P.			
4 .. 0	$\frac{1}{3}$	37	7	5			Multiplicand.
4 .. 0		4	8	6			Multiplier.
		150	5	8	S.		
I. P.		12	6	5	8		
0 .. 6	$\frac{1}{8}$	12	6	5	8	T.	
		1	6	9	8	6	
Product		177	1	5	0	6	

In this example I first multiply by 4 feet as usual.—Then for the 8 inches I say 4 inches is the third of a foot, therefore I take the third part of 37 feet 7 inches 5 parts, which is 12 feet 6 inches 5 parts 8 seconds, and set it down twice. Then for 6 parts, I say, 6 parts are the eighth of 4 inches, because 12 parts make 1 inch; hence it follows, that whatever be the value, or product, by 4 inches, the value of 6 parts will be one-eighth thereof; therefore I take one-eighth of 12 feet 6 inches 5 parts 8 seconds, and find it to be 1 foot 6 inches 9 parts 8 seconds 6 thirds; so that the sum of the whole is 177 feet 1 inch 5 parts 6 thirds.

RULE III.

When the Feet both in the Multiplicand and Multiplier are large numbers.

Multiply the feet *only* into each other: Then, for the inches and parts in the multiplier, take parts of the feet, inches, &c. of the multiplicand: And, for the inches and parts of the multiplicand, take parts of the feet *only* in the multiplier. The sum of all will be the product.

EXAMPLE 5. Let 75 feet 9 inches be multiplied by 17 feet 7 inches.

I.		F.	I.	P.
6	$\frac{1}{2}$	75 ..	9	
		17 ..	7	
		525		
		75.		
1	$\frac{1}{6}$	37 ..	10 ..	6
		6 ..	3 ..	9
		12 ..	9 ..	
Product		1331 ..	11 ..	3

I.		F.	I.
6	$\frac{1}{2}$	17 ..	
3	$\frac{1}{2}$	8 ..	6
		4 ..	3
		12 ..	9

In this example, because there are more than 12 feet in the *multiplier*, I first multiply the 75 feet by 17 feet. Then, I say, 6 inches are the half of a foot, and take the half of 75 feet 9 inches, which is 37 feet 10 inches 6 parts; but I ought to take the parts for 7 inches, therefore I say 1 inch is the sixth of 6 inches, and take the sixth part of 37 feet 10 inches 6 parts, which I find to be 6 feet 3 inches 9 parts. Then, because there are 9 inches in the *multiplicand*, I take parts with them out of the 17 feet in the multiplier, saying, 6 inches are the half of a foot, I there-

fore take the half of 17 feet, which is 8 feet 6 inches; again, because I have 3 inches left, and whatever the product by 6 inches may be, that by 3 inches must be the half thereof; I say 3 inches are the half, and take the half of 8 feet 6 inches, which is 4 feet 3 inches, the sum of these is 12 feet 9 inches, which I place under the former parts, and the sum of the whole is 1331 feet 11 inches 3 parts.

Example 6. Let 311 feet 4 inches 7 parts be multiplied by 36 feet 7 inches 5 parts.

In. P.		F. I. P.
6 ..	$\frac{1}{2}$	311 .. 4 .. 7
		36 .. 7 .. 5
		1866
		933. I. P. S.
	-	155 .. 8 .. 3 .. 6
1 .. 0	$\frac{1}{6}$	25 .. 11 .. 4 .. 7 T.
0 .. 4	$\frac{1}{3}$	8 .. 7 .. 9 .. 6 .. 4
0 .. 1	$\frac{1}{4}$	2 .. 1 .. 11 .. 4 .. 7
		13 .. 9
Product		11402 .. 2 .. 4 .. 11 .. 11

I. P.		F.
4 .. 0	$\frac{1}{3}$	36
0 .. 6	$\frac{1}{8}$	12
0 .. 1	$\frac{1}{6}$	1 .. 6
		0 .. 3
		13 .. 9

In this example I first multiply the feet as in example 5th. Then I say 6 inches are the half of a foot, and take the half of 311 feet 4 inches 7 parts, which I find to be 155 feet 8 inches 3 parts 6 seconds; then, as 1 inch is one-sixth of 6 inches, I therefore take one sixth of 155 feet 8 inches 3 parts 6 seconds, which is 25 feet 11 inches 4 parts 7 seconds: Then, because 4 parts are one third of an inch, I take one third of 25 feet 11 inches 4 parts 7 seconds, and find it to be 8 feet 7 inches 9 parts 6 seconds 4 thirds; and as I have one part left, I say 1 is the fourth of 4, and take the fourth of 8 feet 7 inches 9 parts 6 seconds 4 thirds, which is 2 feet 1 inch 11 parts 4 seconds 7 thirds.—Then for

the four inches in the *multiplicand*, I take a third part of 36, the *feet* in the *multiplier;* because 4 inches are one-third of a foot; and 6 parts are the eighth of 4 inches, I therefore take the eighth of 12 feet which is 1 foot 6 inches; then I have 1 part left, which is the sixth of 6 parts, so I take the sixth of 1 foot 6 inches, which is 3 inches. The sum of these parts is 13 feet 9 inches, which I place under the former parts, and add them together, so that the whole is 11402 feet 2 inches 4 parts 11 seconds 11 thirds.

PRACTICAL EXAMPLES.

7. Let 97 feet 8 inches be multiplied by 8 feet 9 inches. *Ans.* 854 feet 7 inches.

8. Let 87 feet 5 inches be multiplied by 35 feet 8 inches. *Ans.* 3117 feet 10 inches 4 parts.

9. Let 259 feet 2 inches be multiplied by 48 feet 11 inches. *Ans.* 12677 feet 6 inches 10 parts.

10 Multiply 179 feet 3 inches by 38 feet 10 inches. *Ans.* 6960 feet 10 inches 6 parts.

11. Multiply 246 feet 7 inches by 46 feet 4 inches. *Ans.* 11425 feet 0 inches 4 parts.

12. Multiply 246 feet 7 inches by 36 feet 9 inches. *Ans.* 9061 feet 11 inches 3 parts.

13. Multiply 257 feet 9 inches by 39 feet 11 inches. *Ans.* 10288 feet 6 inches 3 parts.

14. Let 8 feet 4 inches 3 parts 5 seconds 6 thirds be multiplied by 3 feet 3 inches 7 parts 8 seconds 2 thirds. *Ans.* 27 feet 7 inches 3 parts 5 seconds 1 third 8 fourths 8 fifths 11 sixths.

15. Multiply 321 feet 7 inches 3 parts by 9 feet 3 inches 6 parts. *Ans.* 2988 feet 2 inches 10 parts 4 seconds 6 thirds.

16 Multiply 42 feet 7 inches 8 parts by 7 feet 3 inches 6 parts. *Ans.* 310 feet 10 inches 10 parts 10 seconds.

17. Multiply 124 feet 7 inches 9 parts by 14 feet 6 inches 2 parts.

Ans. 1809 feet 1 inch 1 part 9 seconds 6 thirds.

18. Multiply 259 feet 10 inches 8 parts by 18 feet 5 inches 4 parts.

Ans. 4793 feet 6 inches 0 parts 10 seconds 8 thirds.

19. Multiply 267 feet 7 inches 10 parts by 25 feet 9 inches 7 parts.

Ans. 6905 feet 0 inches 5 parts 0 seconds 10 thirds.

20 Multiply 317 feet 9 inches 7 parts by 37 feet 5 inches 9 parts.

Ans. 11910 feet 9 inches 11 parts 1 second 3 thirds.

CHAPTER X.

Explanation of the line of numbers on GUNTER'S SCALE, *and the construction and use of the* COMMON DIAGONAL SCALE.

THE line of numbers on the two feet Gunter's Scale, marked *Number*, is numbered from the left-hand of the Scale towards the right, with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9; 1, which stands exactly in the middle of the Scale; the numbers then go on 2, 3, 4, 5, 6, 7, 8, 9; 10, which stands at the right-hand end of the Scale.

These two *equal* parts of the Scale are also *equally divided*, the distance between the first, or left-hand 1, and the first 2, 3, 4, &c. is exactly equal to the distance between the middle 1, and the numbers 2, 3, 4, &c. which follow it.

The subdivisons of these two equal parts of the scale are likewise similar, *viz.* they are each one tenth of the primary divisions, and are distinguished by lines of about half the length of the primary divisions. These subdivisions are again divided into ten parts, where room will admit, and where that is not the case, the units must be estimated, or guessed at by the eye, which is easily done by a little practice.

The primary divisions on the second part of the scale are estimated according to the value set upon the unit on the left-hand of the scale: Thus, if you call the unit on the left-hand of the scale 1, then the first 1, 2, 3, 4, &c. stand for 1, 2, 3, 4, &c. the middle 1 is 10; and the 2, 3, 4, &c. following, stand for 20, 30, 40, &c. and the 10 at the right-hand is 100.—If you call the unit on the left-hand of the scale 10, then the first 1, 2, 3, 4, &c. stand for 10, 20, 30, 40, &c. the middle 1 will be 100; and the 2, 3, 4, &c. following will be 200, 300, 400, &c. and the 10 at the right-hand will be 1000—If you call the unit on the left-hand of the scale, 100, then the first 1, 2, 3, 4, &c. will stand for 100, 200, 300, 400, &c. the middle 1 will be 1000; and the 2, 3, 4, &c. following, 2000, 3000, 4000, &c. and the 10 at the right-hand end will be 10000.—Lastly, if you consider the unit on the left-hand of the scale as one-tenth of an unit, then the first 1, 2, 3, 4, &c. will be $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c. the middle 1 will stand for an unit, and the 2, 3, 4, &c. following it will be 2, 3, 4, &c. and the 10 at the right-hand end of the scale will stand for 10.

From the above description it will be easy to find the divisions representing any given number. Suppose 12 was required; take the division at the figure 1, in the middle of the scale, for the first figure of 12; then for the second figure, count two of the longer strokes to the right-hand, and this last is the point representing 12, where there is a brass pin.—If 34 were required: Call the figure 3 on the right-hand

half of the scale, 30, and count forward four of the longer divisions towards the right-hand; if 340 were required, it must be found in the same manner. If the point representing 345 were required, find 340 as above, then the middle distance between the point of 340, and the point representing 350, will be the point representing 345.

By the line of numbers and a pair of compasses almost all the problems in mensuration may be readily done, for they in general depend upon proportion.—And, as in natural numbers, the quotient of the first term, of any abstract proportion, by the second, is equal to the quotient of the third term by the fourth; so in logarithms (for the line of numbers is a logarithmical line) the difference between the first and second term, is equal to the difference between the third and fourth; consequently, on the line of numbers, the distance between the first and second term, will be equal to the distance between the third and fourth.—And for a similar reason, because four proportional quantities are alternately proportional, the distance between the first and third term will be equal to the distance between the second and fourth. Hence the following

GENERAL RULE.

The extent of the compasses from the first term to the second, will reach, in the same direction, from the third to the fourth: Or, the extent of the compasses from the first term to the third, will reach in the same direction, from the second to the fourth.

By the same direction must be understood, that if the second term lie on the right-hand of the first, the fourth term will lie on the right-hand of the third, and the contrary. Hence,

I. *To find the product of two Numbers.*

As an unit is to the multiplier, so is the multiplicand to the product.

II. *To divide one Number by another.*

As the divisor is to the dividend, so is an unit to the quotient.

III. *To find a mean proportional between two Numbers.*

Because the distance between the first and second term, is equal to the distance between the third and fourth; therefore, if you divide the space between the point representing the first term, and that representing the fourth, into two equal parts: the middle point must necessarily give the mean proportional sought.

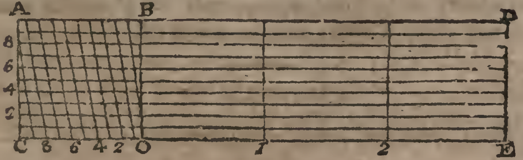
IV. *To extract the Square Root.*

The square root of a quantity is nothing more than a mean proportional between an unit and the given number to be extracted; the unit being the first term, and the number to be extracted the fourth; therefore it may be done by the preceding direction.

NOTE. These rules are all applied in the succeeding parts of the book.

Of the Diagonal Scale.

The diagonal scale, usually placed on the two feet *Gunter's* scale, is thus contracted.



Draw eleven lines of equal length and at equal distances from each other, as in the above figure; divide the two outer lines, as AD, CE, into any convenient number of equal parts, according to the largeness you intend your scale. Join these parts by straight lines, as AC, B*o*, &c. first taking care that the corners C, A, D, E, are all square. Again divide the

lengths AB, and C*o*, into 10 equal parts, join these parts by diagonal lines, *viz.* from the point A to the first division in C*o*, and from the first division in AB, to the second division in C*o*, &c. as in the figure, and number the several divisions.

The chief use of such a scale as this, is to lay down any line from a given measure; or to measure any line, and thereby to compare it with others. If the large divisions in *o*E be called units, the small divisions in C*o* will be 10ths, and the divisions in the altitude *o*B will be 100th parts of an unit. If the large divisions be tens, the others will be units, and tenth parts. If the large divisions be hundreds, the others will be tens and units, &c. each set of divisions being tenth parts of the former ones.

For example, suppose it were required to take off 244 from the scale: fix one foot of the compasses at 2 of the larger divisions in *o*E, and extend the other to the number 4 in C*o*; then move both points of the compasses by a parallel motion, till you come at the fourth long line, taking care to keep the right-hand point in the line marked 2; then open the compasses a small matter, till the left-hand foot reaches to the intersection of the two lines marked 4, 4, and you have the extent of the number required. In a similar manner any other number may be taken off.

CHAPTER XI.

Description and use of the common C*ARPENTER'S* R*ULE.*

THIS rule is generally used in measuring of timber, and artificers works: and is not only useful in taking dimensions, but in casting up the contents of such work.

It consists of two equal pieces of box, each one foot in length, connected together by a folding joint; in one of these equal pieces there is a slider, and four lines

marked at the right-hand A, B, C, D; two of these lines, B, C, are upon the slider, and the other two, A, D, upon the rule. Three of these lines, *viz.* A, B, C, are called double lines, because they proceed from 1 to 10 twice over; these three lines are all exactly alike, both in numbers and division. They are numbered from the left-hand towards the right 1, 2, 3, 4, 5, 6, 7, 8, 9, 1 which stands in the middle; the numbers then go on, 2, 3, 4, 5, 6, 7, 8, 9, 10 which stands at the right-hand end of the rule. These numbers have no determinate value of their own, but depend upon the value you set on the unit at the left-hand of this part of the rule; thus if you call it 1, the 1 in the middle will be 10, the other figures which follow will be 20, 30, &c. and the 10 at the right-hand end will be 100. If you call the first, or left-hand unit 10, the middle 1 will be 100, and the following figures will be 200, 300, 400, &c. and the 10 at the right-hand end will be 1000. Or, if you call the first, or left-hand unit, 100, the middle 1 will be 1000, and the following figures 2000, 3000, 4000, &c. and the 10 at the right-hand 10,000. Lastly, according as you alter, or number, the large divisions, so you must alter the small divisions proportionably.

The fourth line D, is a single line, proceeding from 4 to 40: it is also called the *girt-line*, from its use in casting up the contents of trees and timber. Upon it are marked WG at 17.15, and AG at 18.95, the wine and ale guage points, to make it serve the purpose of a guaging-rule.

The use of the double lines A and B, is for working the rule of proportion, and finding the areas of plane figures. And the use of the *girt-line* D, and the other double line C, is for measuring of timber.—On the other part of this side of the rule, there is a table of the value of a load, or 50 cubic feet, of timber, at all prices, from 6 pence to 24 pence, or two shillings, per foot.

On the other side of the rule are several plane scales divided into 12th parts, marked *inch*, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, &c. signifying, that the inch, $\frac{3}{4}$ inch, &c. are each divided into 12 parts. These scales are useful for planning dimensions that are taken in feet and inches. The edge of the ruler is divided into inches, and each of these inches into eight parts, representing half inches, quarter inches, and half quarters.

In this description we have supposed the rule to be folded; let it now be opened, and slide out the slider, you will find the back part of it divided like the edge of the rule, so that altogether will measure 1 yard or 3 feet in length.

Some rules have other scales and tables upon them; as a table of board measure, one of timber measure; a line for shewing what length for any breadth will make a foot square; also a line shewing what length for any thickness will make a solid foot; the former line serves to complete the table of board measure, and the latter the table of timber measure.

The thickness of a rule is generally about a quarter of an inch; this face is divided into inches, and tenths, and numbered, when the rule is opened, from the right-hand towards the left, 10, 20, 30, 40, &c. towards 100, which falls upon the joint; the other half is numbered in the same manner, and the same way. This scale serves for taking dimensions, in feet, tenths, and hundredths of a foot, which is the most commodious way of taking dimensions, when the contents are cast up decimally.

The use of the Sliding Rule.

PROBLEM I.

To multiply Numbers together, as 12 *and* 16.

Set one on B. to the multiplier (12) on A; then against the multiplicand (16) on B, stands the product (192) on A.

II. *Find the product of* 35 *and* 19.

Set 1 on B, to the multiplicand (35) on A; then because 19 on B runs beyond the rule, I look for 1.9 on B, and against it on A, I find 66.5; but the real multiplier was divided by 10, therefore the product 66.5 must be multiplied by 10, which is done by taking away the decimal point, so the product is 665.

PROBLEM II.

To divide one Number by another, as 360 *by* 12.

Set the divisor (12) on A, to 1 on B; then against the dividend (360) on A, stands the quotient (30) on B.

II. *Divide* 7680 *by* 24.

Set the divisor (24) on A, to 1 on B; then because 7680 is not contained on A, I look for 768 on A, and against it I find 32 on B, the quotient; but because one-tenth of the dividend was taken to make it fall within the compass of the scale A, the quotient must be multiplied by 10, which gives 320.

PROBLEM III.

To Square any Number, as 25.

Set 1 upon C to 10 upon D—Then observe, that if you call the 10 upon D, 1, the 1 on C will be 1; if you call the 10 on D, 10, then the 1 on C will be 100; if you call the 10 on D, 100, then the 1 on C will be 1000, &c. This being well understood, you will observe that against every number on D, stands its square on C.

Thus against 25 stands		625
	against 30 stands	900
	against 35 stands	1225
	against 40 stands	1600

Reckoning the 10 on D, to be 10.

PROBLEM IV.

To extract the Square Root of a Number.

Fix the slider exactly as in the preceding problem, and estimate the value of the lines D and C in the same manner; then against every number found on C stands its square root on D.

PROBLEM V.

To find a mean Proportional between two given Numbers, as 9 and 25.

Set the one number (9) on C, to the same (9) on D; then against 25 on C, stands 15 on D, the mean proportional sought.

For, 9 : 15 :: 15 : 25.

2. What is the mean proportional between 29 and 430?

Set 29 on C, to 29 on D; this being done, you will find that 430 on C will either fall beyond the scale D, or it will not be contained on C. Therefore take the 100th part of it, and look for 4.3 on C, and against it on D stands 11.2, which multiply by 10, and 112 is the mean proportional required.

PROBLEM VI.

To find a fourth Proportional to three Numbers; or, to perform the Rule of Three.

Suppose it were required to find a fourth proportional to 12, 28, and 57.—Set the first term (12) on B to the second (28) on A; then against the third term (57) on B stands the fourth (133) on A.

If either of the middle numbers fall beyond the line, take one-tenth part of that number, and increase the answer 10 times.

Note. The finding a third proportional between two numbers is exactly the same, for in such cases the second number is repeated to form the third: Thus the third proportion to 12 and 28 is 65.33, and is thus found 12 : 28 :: 28 : 65.33. *viz.* set 12 on B, to 28 on A; then against 28 on B stands 65.3 on A.

CHAPTER XII.

Practical Geometry.

DEFINITIONS.

1. GEOMETRY is a science which teaches and demonstrates the properties, affections, and measures of all kinds of magnitude, or extension; as solids, surfaces and lines.

2. Geometry is divided into two parts, theoretical and practical. Theoretical geometry treats of, and considers, the various properties of extension abstractedly; and practical geometry applies these considerations to the various purposes of life.

3. *A solid* is a figure, or body, of three dimensions, *viz.* length, breadth and thickness. And its boundaries are superficies, or surfaces. Thus A represents a solid.

4. *A superficies*, or surface, is an extension of two dimensions, *viz.* length and breadth, without thickness: And its boundaries are lines. Thus B represents a surface.

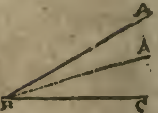
5. *A line* is length without breadth, and is formed by the motion of a point. Thus CD represents a line. Hence the extremities of lines are points, as are likewise their intersections.

6. *Straight lines* are such as cannot coincide in two points without coinciding altogether, and a straight line is the shortest distance between two points.

7. *A point* is position, without magnitude.

8. *A plain rectilineal angle* is the inclination, or opening, of two right lines meeting in a point, as E.

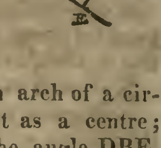
9. One angle is said to be less than another, when the lines which form it are nearer to each other. Take two lines AB and BC touching each other in the point B: Conceive these two lines to open, like the legs of a pair of compasses, so as always to remain fixed to each other in B.—While the extremity A moves from the extremity C the greater is the opening or angle ABC; and on the contrary, the nearer you bring them together, the less the opening or angle will be.



10. *A circle* is a plane figure contained by one line called the circumference, which is every where equally distant from a point within it, called the centre, as C: And an arch of a circle is any part of its circumference, as GH.

11. The magnitude of an angle does not consist in the length of the lines which form it, but in their opening or inclination to each other. Thus the angle ABC is less than the angle DBE, though the lines AB and CB, which form the former angle, are longer than the lines DB and BE, which form the latter.

12. When an angle is expressed by three letters, as ABC, the middle letter always stands at the angular point, and the other two letters at the extremities of the lines which form the angle. Thus the angle ABC is formed by the lines AB and BC, that of DBE is formed by the lines DB and BE.

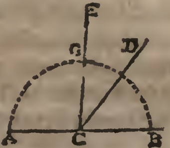


13. Every angle is measured by an arch of a circle, described about the angular point as a centre; thus the arch DE is the measure of the angle DBE, and the arch GH is the measure of the angle ABC.

14. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds.

Angles are measured by the number of degrees cut off from the circle, by the lines which form the angles. Thus, if the arch GH contain 20 degrees, or the eighteenth part of the circumference of the circle, the measure of the angle ABC is said to be 20 degrees.

15. When a right line EC standing upon a right line AB, makes the adjacent angles ACE and BCE equal to each other; each of these angles is said to be a *right angle*, and the line EC is *perpendicular* to AB. The measure of a right angle is therefore 90 degrees, or the quarter of a circle.

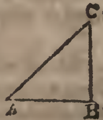


16. *An acute angle* is less than a right angle, as DCB, or ECD.

17. *An obtuse angle* is greater than a right angle as ACD.

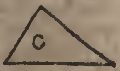
18. *A plane triangle* is a space included by three straight lines, and contains three angles.

19. *A right angled triangle* is that which has one right angle in it, as ABC. The side AC, opposite the right angle, is called the hypothenuse, the side BC is called the perpendicular, and the line AB, on which the triangle stands, is called the base.



20. *An obtuse angled triangle* has one obtuse angle in it, as B.

21. *An acute angled triangle* has all its three angles acute, as C.



22. *An equilateral triangle* is that which has three equal sides, and three equal angles, as D.

23. *An isosceles triangle* has two equal sides, and the third side either greater or less, than each of the equal sides, as E.

24. *A scalene triangle* has all its three sides uneqal, as F.

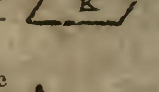
25. *A quadrilateral figure* is a space included by four straight lines, and contains four angles.

26. *A parallelogram* is a plane figure bounded by four right lines, whereof those which are opposite are parallel one to the other; that is, if produced ever so far would never meet.

27. *A square* is an equilateral parallelogram, *viz.* having all its sides equal, and all its angles right angles, as G.

28. *An oblong* is a rectangled parallelogram, whose length exceeds its breadth, as H.

29. *A rhombus* is a parallelogram having all its sides equal, but its angles are not right angles, as I.



30. *A rhomboides* is a parallelogram having its opposite sides equal, but its length exceeds its breadth, and its angles are not right angles, as K.

31. *A trapezium* is a plane figure contained under four right lines, no two of which are parallel to each other, as L. A line connecting any two opposite angles of the trapezium, as AB, is called a diagonal.

32. *A trapezoid* is a plane quadrilateral figure, having two of its opposite sides parallel, and the remaining two not parallel; as M.

33. *Multilateral* figures, or polygons, have more than four sides, and receive particular names, according to the number of sides. Thus, a *Pentagon*, is a Polygon of five sides; a *Hexagon*, has six sides; a *Heptagon*, seven; an *Octagon*, eight; a *Nonagon*, nine; a *Decagon*, ten; an *Undecagon*, eleven; and a *Duodecagon* has twelve sides. If all the sides and angles are equal, they are called regular polygons; if unequal, they are called irregular Polygons, or Figures.

34. *The diameter* of a circle is a right line drawn through the centre, and terminated by the circumference both ways; thus AB is a diameter of the circle. The diameter divides the surface, and circumference, into two equal parts, each of which is called a *semicircle.* If a line CD be drawn from the centre C perpendicular to AB, to cut the circumference in D, it will divide the semicircle into two equal parts, ACD and DCB, each of which will be a *quadrant*, or one-fourth of the circle. The line CD, drawn from the centre to the circumference, is called the *radius.*

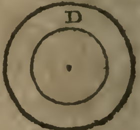
35. A *sector* of the circle is comprehended under two *radii*, or semidiameters, which are supposed not to make one continued line, and a part of the circumference. Hence a sector may be either less or greater, than a semicircle; thus ACB is a sector less than a semicircle, and the remaining part of the circle is a sector greater than a semicircle.

36. *The chord* of an arch is a right line less than the diame-

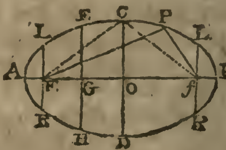
ter, joining the extremities of the arch; thus DE is the chord of the arch DGE, or of the arch EBAD.

37. *A segment* is any part of a circle bounded by an arch and its chord, and may be either greater or less than a semicircle.

38. *Concentric circles* are those which have the same centre, and the space included between their circumferences is called a ring, as D.



39. If two pins be fixed at the points F, *f*, and a thread FP*f* be put round them and knotted at P: then if the point P and the thread be moved about the fixed centres F, *f*, so as to keep the thread always stretched, the point P will describe the curve PBDACP called an *ellipsis*.



40. The points or centres F, *f*, are called the *foci*, and their distance from C, or D, is equal to the half of AB.

41. The line AB, drawn through the *foci* to the curve, is called the *transverse* axis, or diameter. The point O, in the middle of the áxis AB, is the centre of the ellipsis.

42. The line CD, drawn through the centre O, perpendicular to the transverse diameter AB, is called the conjugate axis, or diameter.

43. The line LR, drawn through the focus F, perpendicular to the transverse axis, is called the *parameter*, or *latus rectum*.

44. A line drawn from any point of the curve, perpendicular to the transverse axis, is called an *ordinate* to the transverse, as EG. If it go quite through the figure, as EH, it is called a *double ordinate*.

45. The extremity of any diameter is called the vertex; thus the vertices of the transverse diameter AB, are the points A and B.

46. That part of the diameter between the vertex and the ordinate is called an *abscissa;* thus GB and AG, are abscisses to the ordinate GE.

47. If one end of a thread equal in length to CD, be fixed at the point F, and the other end fixed at D, the end of the square BCD; and the side CB of the square be moved along the right line AB, so as always to coincide therewith, the string being kept stretched and close to the side of the square PD, the point P will describe a curve HROLPG, called a parabola.

48. The fixed point F is called the focus.

49. The right line AB is called the directrix.

50. The line ON is the axis of the parabola, and O is the vertex.

51. A line LR, drawn through the focus F, perpendicular to the axis, is called the parameter, or latus rectum.

52. A right line IK, drawn from the curve perpendicular to the axis, or parallel to the directrix, is called an ordinate; if it go quite through the figure, as IM it is called a double ordinate.

53. The part of the axis, KO, between the vertex, O, and ordinate, IK, is called the abscissa.

Note. The definitions of the solids are given in Part II. Chap. II. in the respective sections where each solid is considered.

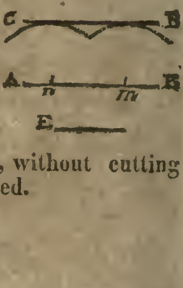
PROBLEM I.

To bisect, or divide a given right line AB, *into two equal parts.*

From the points A and B with any radius, or opening of the compasses, greater than half AB, describe two arches cutting each other in C and D; draw CD, and it will cut AB in the point E, making AE equal to EB.

PROBLEM II.

At a given distance E, *to draw a right line* CD *parallel to a given right line* AB.



From any two points *m n* in the line AB with the extent of E in your compasses describe two arches *o s*;—draw CD to touch these arches, without cutting them, and it will be parallel as required.

PROBLEM III.

Through a given point o, *to draw a right line* CD, *parallel to a given straight line* AB.

Take any point *m* at pleasure in the line AB; with *m* as a centre and distance *m o*, describe the arch *o n*; with *n* as a centre and the same distance describe the arch *sm*. Take the arch *on* in your compasses, and apply it from *m* to *s*; through *o*, *s*, draw CD, and it will be parallel as required.

PROBLEM IV.

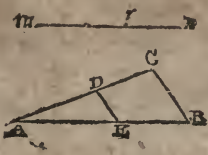
To divide a given right line AB *into any number of equal parts.*

From the ends A and B of the given line, draw two lines AP and BK of any length, parallel to each other. Then set any number of equal parts from A towards P, and likewise from B towards K; draw lines between the corresponding points HM, GL, FI, &c. and they will divide AB into the equal parts AC, CD, DE EB.

PROBLEM V.

To divide a given right line. AB *into two such parts, as shall be to each other, as* mr, *to* rn.

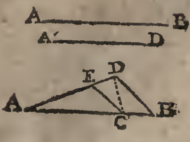
From A draw any line AC, equal to *mn*, and upon it transfer the divisions of the line *mn*, viz. *mr* and *rn*. Join BC, and parallel to it draw DE; then will AE : EB :: *mr* : *rn*.



PROBLEM VI.

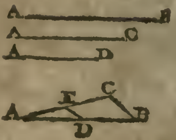
To find a third Proportional to two given lines AB, AD.

Place the two given lines B and AD so as to make an angle with each other at A; in AB the greater, cut off a part AC, equal to AD the less given line; join BD, and draw CE parallel to it, then will AE be the third proportional required, *viz.* AB : AD :: AD : AE.



PROBLEM VII.

To find a fourth Proportional to three given lines. AB, AC, AD.

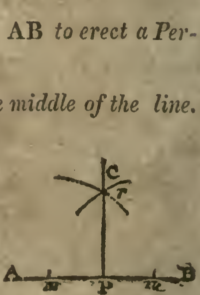


Place two of the given lines AB and AC so as to make an angle with each other at A, and join BC. On AB, set off the distance AD, and draw DE parallel to BC; then AE is the fourth proportional required, *viz.* AB : AD :: AC : AE.

PROBLEM VIII.

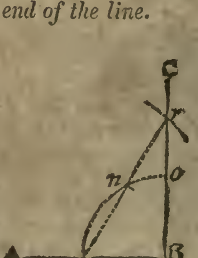
From a given point P *in a right line* AB *to erect a Perpendicular.*

1. *When the point is in, or near, the middle of the line.*



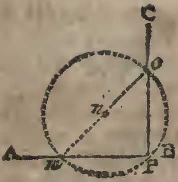
On opposite sides of the point P take two equal distances Pm, Pn from the points m and n as centres, with any opening of the compasses greater than Pm, describe two arches cutting each other in r; through r, draw CP, and it will be the perpendicular required.

2. *When the point P is at the end of the line.*



With the centre P and any radius describe the arch $m\ n\ o$; set off Pm from m to n, and from n with the same radius describe an arch r: through m and n draw the line $m\ n\ r$ to cut the arch in r; then through r and P draw CP and it will be the perpendicular required.

OR THUS:

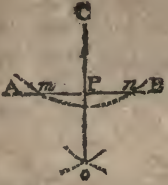


Set one foot of the compasses in P and extend the other to any point n, out of the line AB; from n as a centre and distance nP, describe a circle cutting AB in m; through m and n, draw $m\ n\ o$ to cut the circle in o, then through o draw CP, which will be perpendicular to AB.

PROBLEM IX.

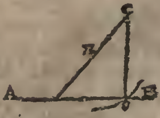
From a given point C *to let fall a Perpendicular upon a given line* AB.

1. *When the point is nearly opposite the middle of the line.*



From the centre C describe an arch to cut AB in m and n; with the centres m and n, and the same radius, describe arches intersecting in o; through C and o draw CP, the perpendicular required.

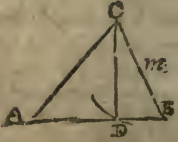
2. *When the point is nearly opposite the end of the line.*



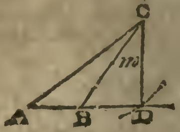
From the given point C draw any line Cm meeting AB in the point m; besect mC in n, and with n as a centre and radius $n\ m$, or nC describe an arch cutting AB in o, draw Co, and it will be the perpendicular required.

PROBLEM X.

In any Triangle ABC *to draw a Perpendicular from any Angle to to its opposite side.*



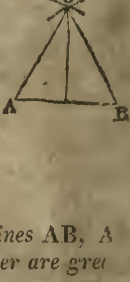
Biseet either of the sides containing the angle from which the perpendicular is to be drawn, as BC in *m*; with the radius *m* B and centre *m* describe an arch cutting AB (produced if necessary) in D, draw CD, and it will be the perpendicular required.



PROBLEM XI.

Upon a given right line AB *to describe an Equilatera Triangle.*

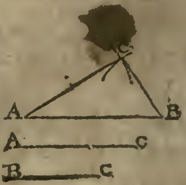
With B as a centre and radius equal to AB describe an arch; with A as a centre and distance, AB, cross it in C; draw AC and BC, then will ABC be the equilateral triangle required.



PROBLEM XII.

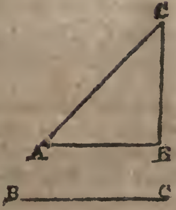
To make a Triangle with three given lines AB, A *and* BC, *of which any two taken together are gre than the third.*

With A as a centre and radius AC, describe an arch, with the centre B and distance BC cross it in C, draw AC, and BC, then ABC is the triangle required.



PROBLEM XIII.

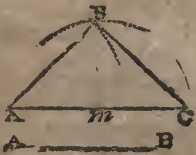
Two sides AB *and* BC *of a right angled Triangle are given, to find the Hypothenuse.*



From the point B in AB draw BC perpendicular and equal to BC: join AC, and it will be the hypothenuse required.

PROBLEM XIV.

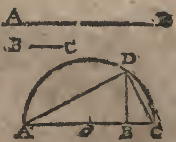
The Hypothenuse AC, *and one side* AB, *of a right angled Triangle are given, to find the other side.*



Bisect AC in *m*, with *m* as a centre and distance *m* A describe an arch, with A as a centre and distance AB cross it in B; join BC: then ABC is a right angled triangle, and BC the required side.

PROBLEM XV.

To find a mean Proportional between two given lines AB *and* BC.



Join AB and BC in one straight line *viz.* make AC equal to the sum of them, and bisect it in the point *o*. With the centre *o* and radius *o* A describe a semicircle; at B erect the perpendicular BD, and it will be the mean proportional required, *viz.*

AB : BD :: BD : BC.

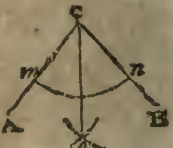
NOTE. If AD and DC be joined, AD will be a mean proportional between AB and AC, also CD will be a mean proportional between BC and AC, *viz.*

AB : AD :: AD : AC
and BC : CD :: CD : AC.

PROBLEM XVI.

To bisect, or divide a given Angle into two equal parts.

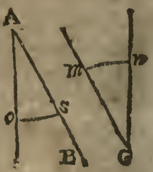
From the centre C with any radius describe an arch *mn*, from *m* and *n* as centres with the same radius describe two arches crossing each other in *o*, draw C*o*, and it will divide the given angle ACB into the two equal angles AC*o* and BC*o*.



PROBLEM XVII.

At a given point A, *in a given line* AB *to make an Angle equal to a given Angle* C.

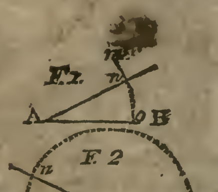
With the centre C and any radius describe an arch *m n*; with the centre A and the same radius describe the arch *os*: Take the distance *m n* in your compasses, and apply it from *s* to *o*; then a line drawn from A through *o* will make the angle A equal to the angle C.



PROBLEM XVIII.

To make an Angle of any proposed number of degrees upon a given right line, by the Scale of Chords.

Upon the line AB to make an angle of 30 deg. (Fig. 1.) Take the extent of 60 degrees from the line of chords with which and the centre A describe the arch *om*. Take 30 degrees from the same scale of chords, and set them off from *o* to *n*; through *n* draw A*n*, then *n*AB is the angle required.



To make an angle of 150 degrees, produce the line BA to *m*, with the centre A and the chord of 60 degrees; describe a semicircle; take the given obtuse angle from 180 degrees, and set off the remainder, *viz.* 30 degrees from *m* to *n*; through *n* draw A*n*, then *n* AB is the angle required.

PROBLEM XIX.

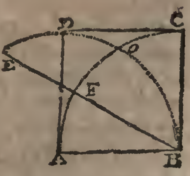
An Angle being given to find how many degrees it contains, by a Scale of Chords.

With the chord of 60 degrees in your compasses and centre A (Fig. 1.) describe the arch *o n*, cutting the two lines which contain the angle in *o* and *n*; take the distance *on* in your compasses, and setting one foot at the beginning of the chords on your scale, observe how many degrees the other foot reaches to, and that will be the number of degrees contained in the arch *o n*, or angle *n*AB.

If the extent *o n* reach beyond the scale, which will always be the case when the angle is obtuse, produce the line BA from A towards *m*, and measure the arch *m n* in the same manner, the degrees it contains, deducted from 180 degrees, will give the measure of the angle *n*AB.

PROBLEM XX.

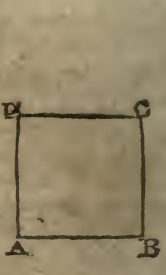
Upon a given right line AB, *to describe a Square.*



With A as a centre and distance AB describe the arch EB, with B as a centre and the same extent describe the arch AC cutting the former in *o*; make *o* E equal to B *o*, and draw BE, make *o*C and *o*D each equal to AF, or F*o*, and join the points AD, DC and CB, then will ABCD be the square required.

OR THUS:

Draw BC perpendicular, and equal to AB; with the extent AB and one foot in A describe an arch; with the same extent and one foot in C cross it in D, join AD and DC, then ABCD is the square required.



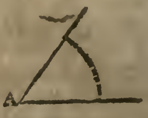
PROBLEM XXI.

To make an Oblong, or Rectangled Parallelogram, of a given length BA, *and breadth* BC.

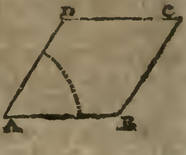
Place BC perpendicular to AB; with the centre A and distance BC describe an arch, with the centre C and distance AB cross it in D; join AD and DC, then ABCD is the oblong required.

PROBLEM XXII.

Upon a given right line AB *to describe a Rhombus, having an angle to the given angle* A.



Upon AB make the angle DAB equal to the given angle A; also make AD equal to AB. Then with D and B as centres and radius AB, describe arches crossing each other in C; join DC and BC, then ABCD is the rhombus required.



PROBLEM XXIII.

To find the centre of a given Circle.

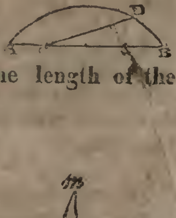
Take any three points, A, C, B, in the circumference of the circle, and join AC and BC. Bisect AC and BC with the lines *no*, and *mo*, meeting each other in the point *o*. Then *o* is the centre required.

Note. By this problem it will be easy to describe a circle through any three given points that are not in the same straight line: Or, to describe a circle about a given triangle. Also, if any arch of a circle be given, the whole circle may be readily described.

PROBLEM XXIV.

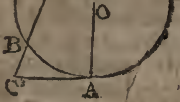
To draw a right line equal to any given arch of a Circle, A B.

Divide the chord AB into four equal parts; set one part AC on the arch from B to D: draw CD, and it will be *nearly* equal to half the length of the arch ADB.



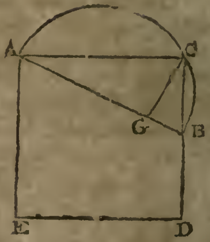
OR THUS:

Through the point A, and *o* the centre of the circle draw A *m*; divide *o n* into four equal parts, and set off three of them from *m* to *n*. Draw AC perpendicular to A *m*, then through *m* and B draw *m* C; then will AC be equal to the length of the arch AB *very nearly*.



PROBLEM XXV.

To make a square equal in Area to a given circle.



Divide the diameter AB into fourteen equal parts, and make AG equal to eleven of these parts; erect the perpendicular GC, and join AC; then the square AEDC, formed upon AC, is equal to the whole circle whose diameter is AB, *exceedingly near the truth.*

PROBLEM XXVI.

In a given circle to describe a Square.

Draw any two diameters AB and CD perpendicular to each other, then connect their extremities, and that will give the inscribed square A DBC.

NOTE. If a side of the square, as DB, be bisected in *m*, and a line *o m n* be drawn from the centre of the circle to cut the circumference: then if the line B*n* be drawn, it will be the side of an octagon inscribed in the circle.

PROBLEM XXVII.

To make a regular Polygon on a given line AB.

Divide 360 degrees by the number of sides contained in your polygon, subtract the quotient from 180 degrees, and the remainder will be the number of degrees in each angle of the polygon. From each end of AB draw lines AO, BO, making angles with the given line equal to half the angle of the polygon: then with the centre O and radius

OA describe a circle to the circumference of which apply continually the given side AB.

OR THUS:

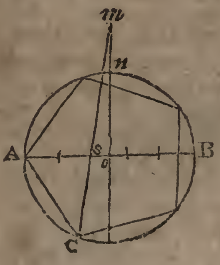
Take the given line AB from any scale of equal parts: multiply the side of your polygon by the number in the third column of the following table, answering to the given number of sides, and the product will give you the length of AO or OB, with which proceed as above.

No. of Sides.	*Name of the Polygon.*	*Rad. of the circumscribing circle.*	*An.* OAB, or OBA.	
3	Trigon,	.5773503	30	*Degrees.*
4	Tetragon,	.7071068	45	
5	Pentagon,	.8506508	54	
6	Hexagon,	1, Side=Radius.	60	
7	Heptagon,	1.1523825	$64\frac{2}{7}$	
8	Octagon,	1.3065630	$67\frac{1}{2}$	
9	Nocagon,	1.4619022	70	
10	Decagon,	1.6186340	72	
11	Undecagon,	1.7747329	$73\frac{7}{11}$	
12	Duodecagon,	1.9318516	75	

PROBLEM XXVIII.

In a given circle to inscribe any regular Polygon; or to divide the circumference of a given circle into any number of equal parts.

Divide the diameter AB into as many equal parts as the figure has sides; from the centre *o* draw the perpendicular *om* divide the radius *on* into four equal parts and set off three of these parts from *n* to *m*, from *m*, through the second division *s*, of the diameter AB draw *m*C: join AC, and it will be the side of the polygon required.



PROBLEM XXIX.

Given the two diameters of an Oval, a figure resembling the conic section called an Ellipsis, to describe it.

Bisect the longer diameter AB in the point C by the line *m n*, bisect the shorter diameter HI in K, set off HK or IK from C towards *m* and *n*: then from A set off A*o* equal to HI, divide the remaining part *o*B into three equal parts, and set off two of them from *o* to S; with S as a centre, and radius SS, cross the line *m n*, in *m* and *n*, through S draw *m* SE, *m* SD, *n*SG, and *n*SF, of an indefinite length.—With the radius AS and centre S describe the arches EAG and FBD; also with *n* as a centre and radius *n*G describe the arch GF, passing through the extremity of the shorter diameter, and meeting BF in the point F; in a similar manner, with *m* as a centre, and radius *m*E or *m*D, describe the arch ED, which will complete the *oval* required.

NOTE. There are various methods of constructing an ellipsis, and oval; the former may be accurately constructed by the 39th and 40th definition, and the latter by the above method. It may be proper here to inform the student, unacquainted with this subject, that it is impossible to describe an ellipsis truly with a pair of compasses; for an ellipsis has no part of the curve of a circle in its composition, an elliptical curve, being described on two points (see definition 39.) is continually varying.

PROBLEM XXX.

Given the Abscissa vm, *and ordinate* Sm, *of a parabola, to construct it.*

Bisect the given ordinate S*m* in B; join B*v* and draw BD perpendicular to it, meeting the abscissa produced in D.

Make *ov* and *v*C, each equal to D*m*, then will *o* be the *focus* of the parabola.

Take any number of points *m*.*m*,&c. in the abscissa, through which draw the double ordinates S*m*S, &c. of an indefinite length. With the radii *o*C, *m*C, &c. and centre *o*, describe arches cutting the corresponding ordinates in the points S, S, &c. and the curve S*v*S, drawn through all the points will be the parabola required.

COMPLETE MEASURER.

PART II.

CHAPTER I.

MENSURATION of SUPERFICIES.

THE area, or superficies, of any plane figure, is estimated by the number of squares contained in its surface; the side of those squares being either an inch, a foot, a yard, a link, a chain, &c. And hence the area is said to be so many square inches, square feet, square yards, square links or square chains, &c. Our common measures of length are given in the first table below, and the second table of square measure is taken from it, by squaring the several numbers.

I. *Lineal Measure.*		II. *Square Measure.*	
12 Inches	1 Foot	144 Inches	1 Foot
3 Feet	1 Yard	9 Feet	1 Yard
6 Feet	1 Fathom	36 Feet	1 Fathom
$16\frac{1}{2}$ Feet / $5\frac{1}{2}$ Yards	1 Pole or Rod	$272\frac{1}{2}$ Feet / $30\frac{1}{4}$ Yards	1 Pole or Rod
40 Poles	1 Furlong	1600 Poles	1 Furlong
8 Furlongs	1 Mile	64 Furlongs	1 Mile

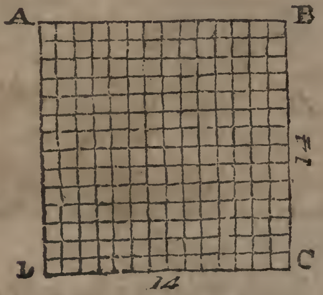
Land is measured by a chain, called *Gunter's chain*, of 4 poles, or 22 yards in length, and consists of 100 equal links, each link being $\frac{22}{100}$ of a yard in length, or 7.92 inches. Ten square chains, or ten chains in

length and one in breadth, make an acre; or 4840 square yards, 160 square poles, or 100,000 square links, each being the same in quantity. Forty perches, or square poles, make a rood, and 4 roods make an acre.

The length of lines measured with a chain, are *generally* set down in links, as whole numbers: every chain being 100 links in length. Therefore after the dimensions are squared, or the superficies is found, it will be in square links; when this is the case, it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. These decimals must be then multiplied by 4 for roods, and the decimals of these again, after five figures are cut off, by 40 for perches.

§ I. *To find the* Area *of a* Square.

Example 1. Let ABCD be a square given, each side being 14. Required the area, or superficial content.



$14 \times 14 = 196$, the area of the square ABCD.

By Scale and Compasses.

Extend the compasses from 1, in the line of numbers, to 14; the same extent will reach from the same point, turned forward, to 196. Or, extend from 10 to 14, that extent will reach to 19.6, which multiply by 10.

Demonstration. Let each side of the given square be divided into 14 equal parts, and lines drawn from one another crossing each other within the square; so shall the whole great square be divided into 196 little squares, as you may see in the figure, equal to the number of square feet, yards, poles, or other measure, by which the side was measured.

2. What is the area of a square whose side is 35.25 chains? *Ans.* 124 acres 1 rood 1 perch.

3. Required the area of a square whose side is 5 feet 9 inches. *Ans.* 33 feet 0 inches 9 parts.

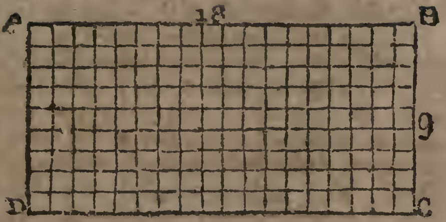
4. What is the area of a square whose side is 3725 links? *Ans.* 138 acres 3 roods 1 perch.

§ II. *Examples of an Oblong, or rectangled Parallelogram.*

To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboid.

RULE.

Multiply the length by the breadth, or perpendicular height, and the product will be the area.



Example 1. Let ABCD be an oblong, the length of it 18 feet, and the breadth 9 feet; these multiply together, the product is 162, the superficial content.

By Scale and Compasses.

Extend the compasses in the line of numbers from 18 to 162, the square feet. Or, call the middle unit on the scale 10, and extend from it to 9, that extent will reach from 18 to 16.2, which multiply by 10.

Demonstration. If the sides AB and CD be each divided into 18 equal parts, representing 18 feet; and the lines AD and BC each divided into 9 equal parts, and lines drawn from point to point, crossing each other within the figure; those lines will make thereby so many little squares as there are square feet, *viz.* 162.

2. What is the superficial content of a rectangular board, whose length is 14 feet 6 inches, and breadth 4 feet 9 inches.

Ans. 68 feet 10 inches 6 parts.

3. Required the area of a rectangular piece of ground, whose length is 1375 links, and breadth 950.

Ans. 13 acres 0 roods 10 perches.

4. What is the superficial content of an oblong which is 2 feet 10 inches 6 parts long, and 9 inches broad?

Ans. 2 feet 1 inch 10 parts 6 seconds.

§ III. *Examples of a Rhombus.*

Example 1. Let ABCD be a rhombus given, whose sides are each 15.5 feet, and the perpendicular EA is 13.42; these multiplied together, the product is 208.010; which is the superficial content of the rhombus, that is, 208 feet and one hundredth part of a foot.

By Scale and Compasses.

Extend the compasses from 1 to 13.42, that extent will reach from 15.5 the same way to 208 feet, the content. Or, call the middle 1 upon the scale 10 and extend from it to 13.42, that extent will reach from 15.5 to 20.8, which multiply by 10.

Demonstration. Let CD be extended out to F, making DF equal to CE, and draw the line BF; so shall the triangle DBF be equal to the triangle ACE: For DF and CE are equal, and BF is equal to AE, because AB and CF are parallel. Therefore the parallelogram ABEF is equal to the rhombus ABCD.

2. What is the area of a rhombus, whose length is 6.2 chains and perpendicular height 5.45 chains?

Ans. 3 acres 1 rood 20.64 perches.

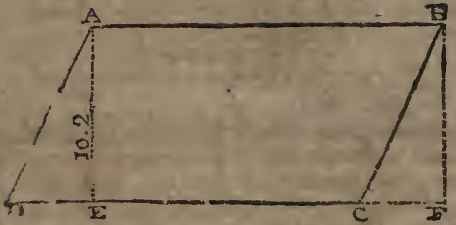
3. The length of a rhombus is 12 feet 6 inches, and perpendicular height 9 feet 7 inches, what is the area?

Ans. 119 feet 9 inches 6 parts.

4. The length of a rhombus is 725 links, and perpendicular height 635 links, what is the area?

Ans. 4 acres 2 roods 16.6 perches.

§ IV. *Examples of a Rhomboides.*



EXAMPLE. 1. Let ABCD be a rhomboides given, whose longest side AB or CD, is 19.5 feet, and the perpendicular AE is 10.2; these multiplied together, the product is 198.9, that is, 198 superficial feet and 9 tenth parts, the content.

DEMONSTRATION. If DC be extended to F, making CF equal to DE, and a line drawn from B to F; so will the triangle CBF be equal to the triangle ADE, and the parallelogram AEFB be equal to the rhomboides ABCD; which was to be proved.

2. A piece of ground, in the form of a rhomboides, measures 4784 links in length, and its perpendicular breadth is 1908 links, what is its area?

Ans. 91 acres 1 rood 4.5952 perches.

3. Required the area of a rhomboides, whose length is 12 feet 6 inches, and perpendicular breadth 5 feet 6 inches. *Ans.* 68 feet 9 inches.

4. How many square yards of painting are in a rhomboides, whose length is 37 feet and breadth 5 feet 3 inches? *Ans.* $21\frac{7}{12}$ yards.

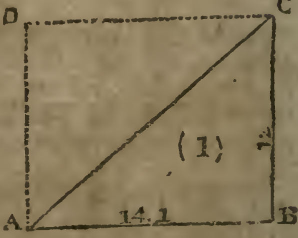
§ V. *To find the Area of a Triangle when the base and perpendicular are given.*

RULE.

Multiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

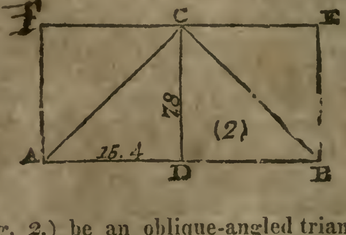
1. Let ABC be a right angled triangle, whose base is 14.1 feet, and the perpendicular 12 feet. Multiply 14.1 by 6, half the perpendicular, and the product is 84.6 feet, the area. Or multiply 14.1 by 12, the product is 169.2; the half of which is 84.6, the same as before.



By Scale and Compasses.

Extend the compasses from 1 to 14.1; that extent will reach the same way from 6 to 84.6 feet, the content. Or, call the middle 1 on the scale 10, extend from it to 14.1, that extent will reach from 6 to 8.46, which multiply by 10.

Note. The perpendicular in the figure ought to be 7.8.



2. Let ABC (*Fig.* 2.) be an oblique-angled triangle given, whose base is 15.4, and the perpendicular 7.8; if 15.4 be multiplied by 3.9 (half the perpendicular,) the product will be 60.06 for the

area, or superficial content: Or, if the perpendicular 7.8 be multiplied into half the base 7.7, the product will be 60.06 as before: Or, if 15.4, the base, be multiplied by the whole perpendicular 7.8, the product will be 120.12, which is the double area; the half whereof is 60.06 feet as before.

By Scale and Compasses.

Extend the compasses from 2 to 15.4, that extent will reach from 7.8 to 60 feet, the content. Or, extend from 20 to 15.4, that extent will reach from 7.8 to 6, which multiply by 10.—Here the middle 1 on the scale is considered as 10.

Demonstration. If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the triangle ADC shall be equal to the given triangle ABC. Hence the parallelogram ABCD is double of the given triangle; therefore half the area of the parallelogram is the area of the triangle. In *Fig.* 2. the parallelogram ABEF is also double of the triangle ABC; for the triangle ACF is equal to the triangle ACD, and the triangle BCE is equal to the triangle BCD; therefore the area of the parallelogram is double the area of the given triangle: Which was to be proved.

3. The base of a triangle is 28.2 yards, and the perpendicular height 18.4 yards, what is the area?

Ans. 259.44 square yards.

4. There is a triangular field whose base measures 1236 links, and perpendicular 731 links; how many acres does it contain?

Ans. 4 acres 2 roods 2.8128 perches.

5. What is the area of a triangle whose base is 18 feet 4 inches, and perpendicular height 11 feet 10 inches? *Ans.* 108 feet 5 inches 8 parts.

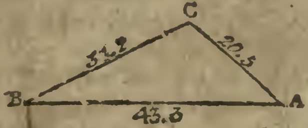
To find the area of any plane Triangle by having the three sides given, without the help of a Perpendicular.

RULE.

Add the three sides together, and take half that sum; from which substract each side severally; then multiply the half sum and the three differences continually, and out of the last product extract the square root; which will be the area of the triangle sought.

EXAMPLES.

1. Let ABC be a triangle, whose three sides are as follows; *viz.* AB 43.3. AC 20.5, and BC 31.2, the area is required.



Sides	43.3	4.2	Differences.
	31.2	16.3	
	20.5	27.0	

Sum 95.0
Half 47.5

Area 296.31

```
  47.5 The half sum.
    27 Difference.
  ----
  3325
  950
  ------
1282.5 Product.
  16.3 Difference.
  ------
   38475
  76950
 12825
 ---------
20904.75 Product.
     4.2 Difference.
 ---------
 4180950
8361900
 ----------
87799.950 Last product.
```

the square root of which is 296.31 (remainder 3339) the area required.

2. What is the area of a triangle whose sides are 50, 40 and 30? *Ans.* 600.

3. The sides of a triangular garden are 41, 29 and 56 yards, what is the area thereof?

Ans. 574.34 yards.

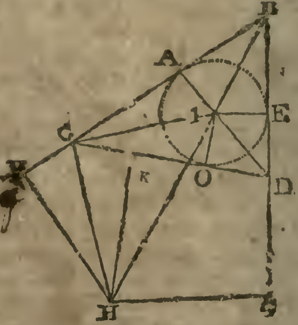
4. How many acres are contained in a triangle whose three sides are 4900, 5025 and 2569 links.

Ans. 61 acres 1 rood 39.68 perches.

5. A field of a triangular form, whose sides are 380, 420 and 765 yards, rents for 55 shillings per acre; what is the annual rent?

Ans. The area is 44699.0347 yards, or 9.235337 acres, which at 55 shillings per acre amounts to £.25 : 7 : 11$\frac{1}{4}$,28, the annual rent.

Demonstration of the rule.—In the triangle BDC, I say, if from the half sum of the sides, you subtract each particular side, and multiply the half sum and the three differences together continually, the square root of the product shall be the area of the triangle.



First, by the lines BI, CI, and DI, bisect the three angles, which lines will all meet in the point I; by which lines the given triangle is divided into three new triangles CBI, DCI, and BDI; the perpendiculars of which new triangles are the lines AI, EI, and OI, being all equal to one another, because the point I is the centre of the inscribed circle (*Euclid, Lib.* IV. *Prob.*. 4.): Wherefore to the side BC join CF equal to DE, or DO; so shall BF be equal to half the sum of the sides; *viz.* $= \frac{1}{2}BC + \frac{1}{2}BD + \frac{1}{2}CD$.

And $BA=BF-CD$; for $CA=CO$ and $OD=CF$; therefore $CD=AF$; and $AC=BF-BD$, for $BE=BA$, and $ED=CF$; therefore $BD=BA+CF$, and $CF=BF-BC$.

Make FH perpendicular to FB, and produce BI to meet it in H. Draw CH and HK perpendicular to CD. Because the angles $FCK+EHK$ are equal to two right angles (for the angles F and K are right angles) equal also to $FCK+ACO$ (by *Euclid*, I. 13.) and the angles $ACO\times AIO$ are equal to two right angles; therefore the quadrangles FCKH and AIOC are similar; and the triangles CIH and AIC are also similar. And the triangles BAI and BFH are likewise similar.

From this explanation, I say, the square of the area of the given triangle; that is, $\overline{BF}^2 \overline{\times IA}^2 = BF \times BA \times CA \times CF$. In words:

The square of BF the half sum of the sides multiplied into the square of IA ($=$I E$=$I O) will be equal to the said half sum multiplied into all the three differences.

For IA : BA :: FH : BF; and IA : CF : : AC : FH; because the triangles are similar. By *Euclid*, *Lib.* VI. *Prop.* 4.

Wherefore multiplying the extremes and means in both, it will be $\overline{IA^2}+BF\times FH=BA\times CA\times CF+FH$; but FH being on both sides of the equation, it may be rejected; and then multiply each part by BF, it will be $\overline{BF^2}\times\overline{IA^2}=BF\times BA\times CA\times CF$. Which was to be demonstrated.

If any two sides of a right angled Triangle be given, the third side may be found by the following

RULE.

1. To the square of the base add the square of the perpendicular, the square root of the sum will give the hypothenuse, or longest side; or from the square of the hypothenuse take the square of either side, and the square root of the remainder, will be the other side.

EXAMPLE 1. Given the base AB (see *Fig.* 1. § V.) 14.1, the perpendicular BC 12, what is the length of the hypothenuse AC ?

```
   14.1 the base AB.          12 the perpend. BC.
   14.1                       12
  ------                     ----
    141                      144 square of BC.
   564
  564
 141
 ------
198.81 square of AB
144.   square of BC.
 ------
```

342.81 Sum of the squares, the square root of which extracted gives AC 18.51513.

2. Given the base 20, and the hypothenuse 25; to find the perpendicular.

From the square of 25=625
Take the square of 15=225

And the square root of 400
will be the perpendicular=20.

3. The base of a right angled triangle is 24, and the perpendicular 18, what is the hypothenuse?
Ans. 30.

4. The wall of a fort standing on the brink of a river is 42.426 feet high, the breadth of the river is 23 yards; what length must a cord be to reach from the top of the fort across the river? *Ans.* 27 yards.

5. The hypothenuse of a right angled triangle is 30, and the perpendicular 18, what is the base?
Ans. 24.

6. A ladder, 50 feet long, will reach to a window 30 feet from the ground on one side of a street; and without moving the foot will reach a window 40 feet high on the other side. The breadth of the street is required. *Ans.* $23\frac{1}{2}$ yards.

7. A line of 380 feet will reach from the top of a precipice, which stands close by the side of a brook, to the opposite bank, and the precipice is 128 feet high, how broad is the brook? *Ans.* 357.29 feet.

8. If a ladder, 50 feet in length, exactly reach the coping of a house, when the foot is 10 feet from the upright of the building; how long must a ladder be to reach the bottom of the second floor window, which is 17.9897 feet from the coping, the foot of this ladder

standing 6 feet from the upright of the building; and what is the height of the wall of the house?

Ans. The height of the wall is 48.9897 feet, and the length of a ladder to reach the second floor window must be 31.5753 feet.

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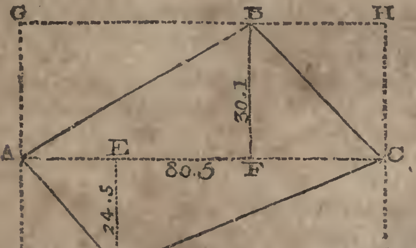
§ VI. *To find the Area of a Trapezium.*

RULE.

Add the two perpendiculars together, and take half the sum, which multiply by the diagonal: the product is the area. Or, find the areas of the two triangles, ABC and ACD (by section V.) and add them together, the sum shall be the area of the trapezium.



EXAMPLES.

1. Let ABCD be a trapezium given, the diagonal of which is 80.5, and the perpendicular BF 30.1, and the perpendicular DE, 24.5; these two added together, the sum is 54.6, the half of which is 27.3 and this being multiplied by the diagonal, 80.5, the product is 2197.65, which is the area of the trapezium.

By Scale and Compasses.

Extend the compasses from 2 to 54.6; that extent will reach from 80.5 to 2197.65, the area. Or call the unit at the beginning of the scale 100, and extend from 200 to 546, that extent will reach from 805 to 2197.65.

DEMONSTRATION. This figure ABCD is composed of two triangles, the triangle ABC is half the parallelogram AGHC: also the triangle ACD is equal to half the parallelogram ACIK, as was proved, sect. V. Wherefore the trapezium ABCD is equal to half the parallelogram GHIK. To find the area, $HI=BF+DE$; therefore $\frac{1}{2}HI\times AC$ ($=KI=GH$) area of the trapezium, which was to be proved.

2. There is a field in the form of a trapezium, whose diagonal is 1660 links, the perpendiculars 702 and 712 links, what is the area?

Ans. 11 acres 2 roods 37.792 perches.

3. What is the area of a trapezium whose diagonal is 34 feet 9 inches, and the two perpendiculars 19 feet 9 inches and 8 feet 9 inches?

Ans. 495 feet 2 inches 3 parts.

4. Required the area of a four-sided field, whose south side is 2740 links, east side 3575 links, north

side 3755 links, west side 4105 links, and the diagonal from south-west to north-east 4835 links.

Ans. 123 acres 0 rood 11.8672 perches.

5. Suppose in the trapezium ABCD (see the foregoing figure) the side AB to be 15, BC 13, CD 14, and AD 12; also the diagonal AC 16; what is the area thereof? *Ans.* 172.5247.

6. Suppose in the trapezium ABCD, on account of obstacles, I could only measure as follows, *viz.* the diagonal AC 378 yards, the side AD 220 yards, and the side BC 265 yards: But it is known that the perpendicular DE will fall 100 yards from A; and the perpendicular BF will fall 70 yards from C; required the area in acres.

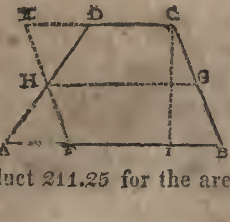
Ans. The perpendicular DE will be 195.959 yards, BF 255.5875 yards; and the area of the trapezium 85342.2885 yards, or 17.6327 acres; or 17 acres, 2 roods 21 perches.

When any two sides of a Trapezium are parallel to each other, it is then generally called a TRAPEZOID; *the area may be found by the following*

RULE.

Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.

EXAMPLES.



1. Let ABCD be a trapezoid, the side AB 23, DC 9.5, and CI 13, what is the area?

First, the sum of AB 23, and DC 9.5, is 32.5; the half whereof is 16.25, which multipled by CI 13, gives the product 211.25 for the area required.

Demonstration. Bisect AD in H, through H draw FE parallel to BC, and then produce CD to E; also draw GH parallel to AB or CD. Now AH is equal to HD, and the angle AHF is equal to the angle EHD (*Euclid*, I. 15.) also the angle HED is equal to the angle HFA (*Euclid*, I. 29.) Therefore the triangles being equiangular, and having one side equal, are equal in all respects (*Euclid*, I. 26.) consequently ED is equal to AF. Again, HG is half the sum of EC and FB, for it is equal to each of them, consequently it is half the sum of AB and DC, but FB multiplied by CI is the area of the figure ECBF, or of its equal ADCB, consequently HG multiplied into CI is the area thereof; which was to be proved.

2. Required the area of a trapezoid whose parallel sides are 750 and 1225 links, and the perpendicular distance between them 1540 links?

Ans. 15 acres 0 roods 33.2 perches.

3. What is the area of a trapezoid whose parallel sides are 12 feet 6 inches, and 18 feet 4 inches; and the perpendicular distance between them 7 feet 9 inches? *Ans.* 119 feet 5 inches 9 parts.

4. A field in the form of a trapezoid, whose parallel sides are 6340 and 4380 yards, and the perpendicular distance between them 121 yards, lets for *l.*83 : 1 : $7\frac{1}{5}$ *per annum*, what is that *per* acre?

*Ans. l.*0 : 12 : $4\frac{4}{5}$.

§ VII. *Of Irregular Figures.*

Irregular figures are all such as have more sides than four, and the sides and angles unequal. All such figures may be divided into as many triangles as there are sides, wanting two. To find the area of such figures, they must be divided into trapeziums and triangles, by lines drawn from one angle to another; and

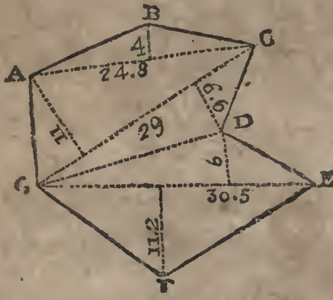
so find the areas of the trapeziums and triangles separately, and then add all the areas together; so will you have the area of the whole figure.

Let ABCDEFG be an irregular figure given to be measured; first, draw the lines AC and GD, and thereby divide the given figure into two trapeziums, ACDG and GDEF, and the triangle ABC; of all which, I find the area separately.

First, I multiply the base AC by half the perpendicular, and the product is 49.6, the area of the triangle ABC.

Then for the trapezium ACDG, the two perpendiculars, 11 and 6.6, added together, make 17.6; the half of which is 8.8, multiplied by 29, the diagonal; the product is 255.2, the area of that trapezium.

And for the trapezium GDEF, the two perpendiculars, 11.2 and 6, added together, make 17.2; the half is 8.6; which multiplied by 30.5, the diagonal, the product is 262.3, the area. All these areas added together, make 567.1, and so much is the area of the whole irregular figure. See the work.



24.8 base AC.
2 half perpendicular.

49.6 area of ABC.

11.2 } perpendiculars.
6. }

17.2 sum.

8.6 half sum.

11 perpendicular.
6.6

17.6 sum.

8.8 half.
29 diagonal CG.

792
176

255.2 area of ACGD.
30.5
8.6

1830
2440

262.30 area of GDEF.
255.2 area of ACDG.
49.6 area of ABC.

567.10 sum of the areas.

This figure being composed of triangles and trapeziums, and those figures being sufficiently demonstrated in the Vth and VIth sections aforegoing, it will be needless to mention any thing of the demonstration in this place.

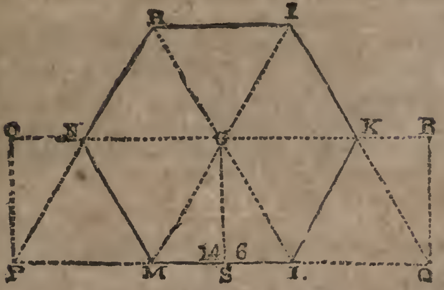
§ VIII. *Of Regular Polygons.*

To find the area, or superficial content, of any regular polygon.

RULE I.

Multiply the whole perimeter, or sum of the sides by half the perpendicular let fall from the centre

to the middle of one of the sides; and the product is the area.



EXAMPLES.

1. Let HIKLMN be a regular hexagon, each side being 14.6, the sum of all the sides is 87.6, the half sum is 43.8, which mmltiplied by the perpendicular GS 12.64397, the product is 553.805886. Or if 87.6, the whole sum of the sides, be multiplied by half the perpendicular 6.321985, the product is 553.805886, which is the area of the given hexagon.

By Scale and Compasses.

Extend the compasses from 1 to 12.64, that extent will reach from 43.8, the same way to 553.8. Or, extend from 2 to 12.64, that extent will reach from

K

87.6 to 553.8. Or, as before, call the first number 100 times as much as it is, and each of the two following 10 times as much as they are.

Demonstration. Every regular polygon is equal to the parallelogram, or long square, whose length is equal to half the sum of the sides, and breadth equal to the perpendicular of the polygon, as appears by the foregoing figure; for the hexagon HIKLMN is made up of six equilateral triangles: and the parallelogram OPQR is also composed of six equilateral triangles, that is, five whole ones, and two halves; therefore the parallelogram is equal to the hexagon.

2. The side of a regular pentagon is 25 yards, and the perpendicular from the centre to the middle of one of the sides is 17.204775; required the area.

Ans. 1075.2984.

3. Required the area of a heptagon, whose side is 19.38, and perpendicular 20.1215.

Ans. 1364.841345.

4. Required the area of an octagon, whose side is 9.941, and perpendicular 12. *Ans.* 477.168.

5. Required the area of a decagon, whose side is 20, and perpendicular 30.776836. *Ans.* 3077.6836.

6. Required the area of a duodecagon, whose side is 102, and perpendicular 190.3345908.

Ans. 116484.7695696.

7. Required the area of a nonagon, whose side is 40, and perpendicular 54.949548. *Ans.* 9890.91864.

8. Required the area of an undecagon, whose side is 20, and perpendicular 34.056874.

Ans. 3746.25614.

9. Required the area of a trigon, *viz.* an equilateral triangle, whose side is 20, and perpendicular 5.773502. *Ans.* 173.20506.

10. Required the area of a tetragon, *viz.* a square, whose side is 20, and perpendicular from the centre to the middle of one of the sides 10. *Ans.* 400.

A TABLE *for the more ready finding the Area of a Polygon, and also the Perpendicular.*

No. of Sides.	*Name of the Polygon.*	I. *Areas. The side* I.	II. *Perpend. The side* I.
3	Trigon,	.433013	.2886751
4	Tetragon,	1.000000	.5000000
5	Pentagon,	1.720477	.6881910
6	Hexagon,	2.598076	.8660254
7	Heptagon,	3.633912	1.0382617
8	Octagon,	4.828427	1.2071068
9	Nonagon,	6.181824	1.3737387
10	Decagon,	7.694209	1.5388418
11	Undecagon,	9.365640	1.7028437
12	Duodecagon,	11.196152	1.8660254

RULE II.

Multiply the square of the side by the tabular area, and the product is the area of the polygon. Or, multiply the side of the polygon by the tabular perpendicular, and the product will give the perpendicular of the polygon; then proceed by the first rule.

How to find the Tabular Numbers.

These numbers are found by trigonometry, thus: find the angle at the centre of the polygon by dividing 360 degrees by the number of sides of the polygon.

EXAMPLE. Suppose each side of the duodecagon annexed be 1, and the area be required.

Divide 360 by 12 (the number of sides) and the quotient is 30 degrees for the angle ACB; the half of which is 15, the angle DCB, whose complement to 90 degrees is 75 degrees, the angle CBD: then say

As s, DCB 15 degrees,	Co-ar. 0.587004
is to .5 the half-side DB log.	9.698970
so is s, CBD 75 degrees,	9.984944
to the perpendicular CD 1.866025	0.270918

Then 1.866025 multiplied by 6, (the half perimeter) the product is 11.196152 the area of the duodecagon required.

EXAMPLE. If the side of a hexagon be 14.6, what is the area?

14.6	2.598076 tab. area.
14.6	213.16
876	15588456
584	2598076
146	7794228
	2598076
213.16 squ.	5196152
	553.80588016 area, the same as at page 97.

NOTE. Should more examples be thought necessary, take any of those under rule I.

§ IX. *Of a Circle.*

There is no figure that affords a greater variety of useful properties than the circle. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter, or has the least perimeter about the same area.

The area of a circle is always less than the area of any regular polygon circumscribed about it, and its circumference always less than the circumference of the polygon. But on the other hand, its area is always greater than that of its inscribed polygon, and its circumference greater than the circumference of its inscribed polygon; hence the circle is always limited between these polygons. The area of a circle is equal to that of a triangle, whose base is equal to the circumference, and perpendicular equal to the radius.

Circles, like other similar plane figures, are in proportion to one another, as the squares of their diameters; and the circumferences of circles, are to one another as their diameters, or radii.

The proportion of the diameter of a circle to its circumference has never yet been *exactly* determined. This problem has engaged the attention, and exercised the abilities of the greatest mathematicians for ages; no square, or any other right-lined figure, has yet been found, that shall be *perfectly* equal to a given circle. But though the relation between the diameter

and circumference has not been accurately expressed in numbers, it may be *approximated* to any assigned degree of exactness. *Archimedes*, about two thousand years ago, discovered the proportion to be nearly as 7 to 22; other, and nearer, ratios have since been successively assigned, *viz.*

As 106 to 333,
As 113 to 355, &c.

This last proportion is very useful, for being turned into a decimal, it agrees with the *truth* to the sixth figure inclusively. *Vieta*, in his *universalium inspectionem ad canonem mathematicum*, published in 1579, by means of the inscribed and circumscribed polygons of 393216 sides, carried the ratio to ten places of figures, shewing that if the diameter of a circle be 1 the circumference will

be greater than 3.1415926535,
but less than 3.1415926537.

And *Ludolf Van Ceulen*, in his book *de circulo et adscriptis*, by the same means carried the ratio to 36 places of figures; this was thought so extraordinary a performance, that the numbers were cut on his tombstone in St. Peter's church-yard, at Leyden. These numbers were afterwards confirmed by Willebrord Snell. Mr. Abraham Sharp, of Little Horton, near Bradford, Yorkshire, extended the ratio to 72 places of figures, by means of Dr. Halley's series, as may be seen in Sherwin's logarithms. Mr. Machin, professor of astronomy in Gresham college, carried the ratio to 100 places of figures; his method may be seen in Dr. Hutton's large treatise on mensuration. Lastly, M. de Lagny, in the *memoirs de l'Acad.* 1719, by means of the tangent of an arch of 30 degrees, has carried the

ratio to the amazing extent of 128 figures; finding that if the diameter be 1, the circumference will be 3.1415, 92653, 58979, 32384, 62643, 38327, 95028, 84197, 16939, 93751, 05820, 97494, 45923, 07816, 49628, 62089, 98628, 03482, 53421, 17067, 98214, 80865, 13272, 30664, 70938, 446+or 447.—But on ordinary occasions 3.1416 is generally used, as being sufficiently exact.

PROBLEM I.

Having the Diameter to find the Circumference, or circumference to find the diameter.

RULES.

1. As 7 is to 22, so is the diameter to the circumference.

Or, as 113 is to 355 so is the diameter to the circumference.

Or, as 1 is to 3.1416, so is the diameter to the circumference.

2. As 22 is to 7, so is the circumference to the diameter.

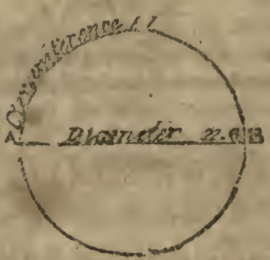
Or, as 355 is to 113, so is the circumference to the diameter.

Or, as 3.1416 is to 1, so is the circumference to the diameter; or, which is the same thing, as 1 is to .318309, so is the circumference to the diameter.

EXAMPLES.

1. The diameter of a circle is 22.6, what is the circumference?

Multiply the diameter 22.6 by 22, the product is 497.2; which divided by 7 gives 71.028 for the circumference. Or, (by the second proportion) if 22.6 be multiplied by 355, the product will be 8023; this divided by 113, the quotient is 71, the circumfercnee. Or (by the third proportion) if 22.6 be multiplied into 3.1416, the product is 71.00016, the circumference: the two last proportions are the most exact.



By Scale and Compasses.

Extend the compasses from 7 to 22; or from 113 to 355, or from 1 to 3.1416; that extent will reach from 22.6 to 71.

2. The circumference of a circle is 71, what is the diameter?

If .318309 be multiplied by 71, the product will be 22.5999239 for the diameter. Or, 113 multiplied by 71, the product is 8023; which divided by 355, the quotient will be 22.6 the diameter: Or 71 multiplied by 7, the product is 497; this divided by 22, the quotient is 22,5999, the diameter.

By Scale and Compasses.

Extend the compasses from 3.1416 to 1, that extent will reach from 71 to 22.6, which is the diameter sought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the same will reach from 71 to 22.6, as before.

NOTE. That if the circumference be 1, the diameter will be .318309; this number being found by dividing an unit by 3.1416.

3. If the diameter of the earth be 7970 miles, what is its circumference, supposing it a perfect sphere? *Ans.* 25038.552 miles.

4. Required the circumference of a circle whose diameter is 50 feet. *Ans.* 157.08 feet.

5. If the circumference of a circle be 12, what is the diameter? *Ans.* 3.819708.

PROBLEM II.

To find the Area of a Circle.

RULES.

1. Multiply half the circumference by half the diameter: Or, take $\frac{1}{4}$ of the product of the whole circumference and diameter, and it will give the area.

2. Multiply the square of the diameter by .7854, and the product will be the area.

3. As 452 is to 355, so is the square of the diameter to the area.

4. As 14 is to 11, so is the square of the diameter to the area.

5. Multiply the square of the circumference by .07958, and the product will be the area.

6. As 88 is to 7, so is the square of the circumference to the area.

7. As 1420 is to 113, so is the square of the circumference to the area.

EXAMPLES.

1. The diameter of a circle is 22.6, and its circumference 71, what is the area by the first rule?

35.5 half the circumference.	22.6 diameter.
11.3 half diameter.	71 circumference.
1065	226
355	1582
355	4|1604.6
401.15 area.	401.15 area.

DEMONSTRATION OF RULE 1. Every circle may be conceived to be a polygon of an infinite number of sides: Now the semidiameter, must be equal to the perpendicular of such a polygon; and the circumference of the circle equal to the periphery of the polygon; therefore half the circumference, multiplied by half the diameter, gives the area. The other part is self-evident.

2. If the diameter of a circle be 1, and its circumference 3.1416, what is the area?

Ans. .7854.

3. If the diameter of a circle be 22.6, what is the area by the second rule?

22.6 diameter.	510.76
22.6	.7854
1356	204304
452	255380
452	408608
	357532
510.76 squ. of the diameter.	401.150904 area.

Demonstration of rule 2. All circles are to each other as the squares of the diameters, and the area of a circle whose diameter is 1, is .7854 (by the second example); therefore as the square of 1, which is 1, is to .7854, so is the square of the diameter of any circle to its area.

4. If the diameter of a circle be 22.6, what is the area by the third rule?

As 452 : 355 :: 510.76 the square of the diameter : 401.15, the same as before.

Demonstration of rule 3. By problem I. we have, as 113 : 355 :: diameter 1 : circumference; but the area of a circle whose diameter is 1, is $\frac{1}{4}$ of this circumference by rule I. problem II. *viz.* it is $\frac{355}{452}$; but the areas of circles are to each other as the squares of their diameters, therefore square 1 : $\frac{355}{452}$: : square of any diameter : the area; *viz.* 452 : 355 : : square of any diameter : the area.

5. If the diameter of a circle be 22.6, what is the area by the fourth rule?

As 14 : 11 :: 510.76 the square of the diameter : 401.31 the area which is larger than by the other methods.

DEMONSTRATION OF RULE 4. By problem I. we have as 7 : 22 :: diameter 1 : circumference; and, by the same argument as above, the area of a circle whose diameter is 1, is $\frac{22}{28}$ or $\frac{11}{14}$; hence, 14 : 11 :: square of the diameter : the area.

6. If the circumference of a circle be 71, what is the area by rule 5?

71 circumference.	.07958
71	5041
71	7958
497	31832
	39790
5041 squ. of the circumference.	401.16278 area.

The reason of the above operation is easy; for as the diameter of one circle is to its circumference, so is the diameter of any other circle to its circumference: therefore the areas of circles are to each other as the squares of their circumferences; and .07958 is the area of a circle whose circumference is 1; for when the circumference is 1, the diameter is .318309, as has been observed before.

7. If the circumference of a circle be 71, what is the area by the sixth rule?

As 88 : 7 :: 5041 the square of the circumference : 400.98 area.

Demonstration of rule 6. By problem I. we have as 22 : 7 :: circumference 1 : diameter; but by rule I. problem II. the area of a circle whose circumference is 1, is $\frac{1}{4}$ of this diameter, *viz.* $\frac{7}{88}$. Now the areas of circles are to each other as the squares of their circumferences; therefore square 1 : $\frac{7}{88}$ :: square of any circumference: area; *viz.* 88 : 7 :: square of any circumference : area.

8. If the circumference of a circle be 71, what is the area by the 7th rule?

As 1420 : 113 :: 5041 the square of the circumference : 401.15 area.

Demonstration of rule 7. By problem I. we have as 355 : 113 :: circumference 1 : diameter; and, by the same argument as above, the area of a circle whose circumference is 1, is $\frac{113}{1420}$; therefore it follows, that 1420 : 113 :: square of any circumference: area.

9. What is the area of a circle whose diameter is 12? *Ans.* 113.0976.

10. The diameter of a circle is 12, and its circumference 37.6992, what is the area? *Ans.* 113.0976.

11. The circumference of a circle is 37.6992, what is the area? *Ans.* 113.104579, &c.

12. The surveying-wheel is so contrived as to turn just twice in length of a pole, or $16\frac{1}{2}$ feet; in going round a circular bowling-green, I observed it to turn exactly 200 times; what is the area of the bowling-green?

Ans. 4 acres 3 roods 35.8 perches.

13. The length of a line, with which my gardener formed a circular fish pond, was exactly $27\frac{3}{4}$ yards: pray what quantity of ground did the fish-pond take up?

Ans. 2419.22895 yards, or half an acre nearly.

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PROBLEM III.

Having the Diameter, Circumference, or Area of a Circle given; to find the side of a Square equal in Area to the Circle, and the side of a Square inscribed in the Circle; Or, having the side of a Square given, to find the Diameter of its Circumscribing Circle, and also of a Circle equal in Area, &c.

RULES.

1. The diameter of any circle, multiplied by .8862269, will give the side of a square equal in area.

2. The circumference of any circle, multiplied by .2820948, will give the side of a square equal in area.

3. The diameter of any circle, multiplied by .7071068, will give the side of a square inscribed in that circle.

4. The circumference of any circle, multiplied by .2250791, will give the side of a square inscribed in that circle.

5. The area of any circle, multiplied by .6366197, and the square root of the product extracted, will give the side of a square inscribed in that circle.

6. The side of any square, multiplied by 1.414236, will give the diameter of its circumscribing circle.

7. The side of any square multiplied by 4.4428829, will give the circumference of its circumscribing circle.

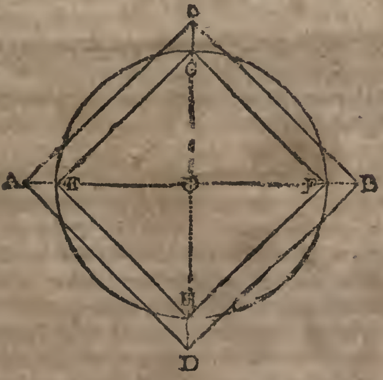
8. The side of any square, multiplied by 1.1283791, will give the diameter of a circle equal in area to the square.

9. The side of any square, multiplied by 3.5449076, will give the circumference of a circle equal in area to the square.

Explanation of these Rules.

Let ADBC represent a square equal in area to the circle: and EHFG a square inscribed in the circle; GH and EF diameters of the circle, perpendicular to each other.

1. The area of a circle, whose diameter is 1, is .7854, or .78539816, &c. the square root of which gives the number in rule 1.—And as the diameter of one circle is to the diameter of another, so is the side of a square equal in area to the former, to the side of a square equal in area to the latter, &c.



2. The area of a circle, whose circumference is 1, is .0795775, &c. the square root of which gives the number in the 2d rule.

3. To the square of $EO = \frac{1}{2}$, add the square of $GO = \frac{1}{2}$, the square root of the sum will give the third number.

4. The diameter of any circle, when the circumference is 1, is .318309, &c. hence EO or GO is 1.159154, &c. with which proceed as above, and you will find the fourth number.

Hence it appears, that if the diameter of the circle is given, or can be found, the side of the inscribed square may be found; and if the side of a square equal in area to the circle is given, the diameter and circumference of the circle may be readily found, the former by dividing the area by .78539, &c. and extracting the square root of the quotient (see rule 2. prob. II.) and the latter by dividing the area by .0795775, &c. and extracting the square root of the quotient (see rule 5. prob. II.)—The rest of the numbers are left for the learner to examine at his leisure.

EXAMPLES.

1. If the diameter of a circle be 22.6, what is the side of a square equal in area to the circle?

The side of a square equal in area to a circle, whose diameter is 1, by rule 1, is .8862269;—hence, $22.6\times 8862269=20.02872794$, the side of the square.

2. If the diameter of a circle be 50, what is the side of a square equal in area? *Ans.* 44.311345.

3. If the circumference of a circle be 40, what is the side of a square equal in area to the circle?

The side of a square equal in area to a circle whose circumference is 1, by rule 2, is .2820948; and $.2820948\times 40=11.283792$, the side of the square.

4. If the circumference of a circle be 71, what is the side of a square equal in area to the circle?

Ans. 20.0287308.

5. If the diameter of a circle be 50, what is the side of a square inscribed in that circle?

The side of a square inscribed in a circle whose diameter is 1, by rule 3, is .7071068, therefore, $50\times .7071068=35.35534$, the side of the square.

6. If the diameter of a circle be 22.6, what is the side of a square inscribed in that circle?

Ans. 15.98061368.

7. If the circumference of a circle be 40, what is the side of a square inscribed in that circle?

The side of a square inscribed in a circle whose circumference is 1, by rule 4, is .2250791; and $40 \times .2250791 = 9.003164$, the side required.

8. If the circumference of a circle be 71, what is the side of a square inscribed in that circle?

Ans. 15.9806161.

9. If the area of a circle be 1963.5, what is the side of a square inscribed in that circle?

The area of a square inscribed in a circle whose area is 1, by rule 5, is .6366197; therefore, $1963.5 \times .6366197 = 1250.00278095$, whose square root is 35.3553, the side of the inscribed square.

10. If the area of a circle be 401.15, what is the side of a square inscribed in that circle.

Ans. 15.9806.

11. If the side of a square be 35.36, what is the diameter of a circle which will circumscribe that square?

The diameter of a circle that will circumscribe a square whose side is 1, by rule 6, is 1.4142136; and $.35.36 \times 1.4142136 = 50.006592896$, the diameter.

12. If the side of a square be 15.98, what is the diameter of a circle which will circumscribe that square. *Ans.* 22.599133328.

13. If the side of a square be 35.36, what is the circumference of a circle which will circumscribe that square?

The circumference of a circle that will circumscribe a square whose side is 1, by rule 7, is 4.4428829 and $35.36 \times 4.4428829 = 157.1000339344$, the circumference.

14. If the side of a square be 15.98, what is the circumference of a circle which will circumscribe that square? *Ans.* 70.997268742.

15. If the side of a square be 35.36, what will be the diameter of a circle having the same area as the square?

The diameter of a circle having the same area as a square whose side is 1, by rule 8, is 1.1283791; hence 35.36×1.1283791=39.899484976, the diameter.

16. If the side of a square be 20.029, what will be the diameter of a circle having the same area as the square? *Ans.* 22.6003049939.

17. If the side of a square be 35.36, what is the circumference of a circle, having the same area as the square?

The circumference of a circle that will be equal to the area of a square whose side is 1, by rule 9, is 3.5449076; therefore—

35.36×3.5449076=125.347932736, the side of the square.

18. If the side of a square be 20.029, what is the circumference of a circle, having the same area as the square? *Ans.* 71.000954.

Note. All the foregoing examples are easily worked by scale and compasses, for they consist only of multiplication. But in the 9th example a square root is to be extracted; and in order to do this right, it will be necessary to call the middle unit upon the scale a square number, as 1, 100, 10000, &c. or 1, $\frac{1}{100}$, &c.—Thus, if the number to be extracted consist of one whole number, call the 1 in the middle of the scale an unit; if of two or three whole numbers, call it 100; if four or five whole numbers, call it 10000, &c. for whole or mixed numbers. For pure decimals, if the number to be extracted consist of one or two places, call the unit in the middle of the scale 1, then the 1 at the left-hand will be $\frac{1}{10}$, the figure 2 will be $\frac{2}{10}$, &c. or the 1 at the left-hand may be called $\frac{10}{100}$, then the figure 2 will be $\frac{20}{100}$, &c. so that you will more readily obtain the intermediate places.—This being understood, extend the compasses from the middle unit thus estimated, to the number to be extracted; divide the space of this extent into two equal parts, and the mid-

dle point will be the root required : but here you must observe that the middle unit will change its value (except you call it an unit), that is, if in the first extent you estimated it at 100, it must now be considered as 10; if at 10000, it must now be called 100, &c. Thus in the 9th example the square root of 1250 is to be extracted; here I consider the middle unit as 10000, and extend from it towards the left-hand to 1250, the middle point between these is 35.3, estimating the middle unit now at 100. A little practice will render these observations familiar.

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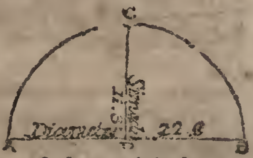
§ X. *To find the Area of a Semicircle.*

RULE.

Multiply the fourth part of the circumference of the whole (that is, half the arch line) by the semidiameter, the product is the area.

EXAMPLES.

1. Let ACB be a semicircle, whose diameter is 22.6, and the half circumference, or arch line, ACB, is 35.5, the half of it is 17.75, which multiply by the semidiameter 11.3, and the product is 200.575, the area of the semicircle.



By Scale and Compasses.

Extend the compasses from 1 to 11.3; that extent will reach from 17.75 to 200.575, the area. Or, as before, call the first number 100 times as much as it is,

and each of the two following 10 times as much as they are.

If only the diameter of the semicircle be given you may say, by the *Rule of Three,*

As 1 is to .3927, so is the square of the diameter to the area; or multiply the square of the diameter by .7854, and take half the product.

2. The diameter of a circle is 30, and the semicircumference 47.124; what is the area of the semicircle? *Ans.* 353.43.

3. The diameter of a circle is 200, what is the area of the semicircle? *Ans.* 15708.

4. If the circumference of a circle be 157.08, and the diameter 50; what is the area of the semicircle? *Ans.* 981.75.

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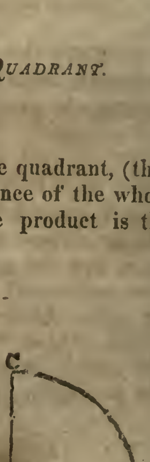
§ XI. *To find the* Area *of a* Quadrant.

RULE.

Multiply half the arch line of the quadrant, (that is, the eighth part of the circumference of the whole circle,) by the semidiameter, and the product is the area of the quadrant.

EXAMPLES.

1. Let ABC be a quadrant, or fourth part of a circle, whose radius, or semidiameter is 11.3, and the half arch line 8.875; these multiplied together, the product is 100.2875 for the area.



These are the rules commonly given for finding the

area of a semicircle and quadrant; but it is the best way to find the area of the whole circle, and then take half that area for the semicircle, and a fourth part for the quadrant.

2. Required the area of a quadrant, the radius being 15, and the length of the arch line 23.562.

Ans. 176.715.

3. The diameter of a circle is 200, what is the area of the quadrant? *Ans.* 7854.

4. If the circumference of a circle be 157.08, and the diameter 50, what is the area of the quadrant?

Ans. 490.875.

Before I proceed to shew how to find the area of the sector, and segment of a circle, I shall shew how to find the length of any arch of a circle.

To find the length of any Arch of a Cirele.

RULE I.

Multiply, continually, the radius, the number of degrees in the given arch, and the number *.01745329; the product will be the length of the arch.

EXAMPLE. 1. If the arch ACB contain 118 degrees 46 minutes 40 seconds, and the radius of the circle, of which ACB is an arch, be 19.9855: what is the length of the arch?

First, 118 degrees 46 minutes 40 seconds, are equal to $118\frac{7}{9}$ degrees.

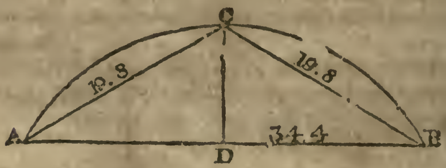
$19.9855 \times 118\frac{7}{9} \times .01745329 = 41.431199$, &c. the length of the arch.

* When the radius is 1, the semi-circumference of the circle is 3.14159265, &c. and this number divided by 180, the degrees in a semicircle, gives .01745329 for the length of one degree when the radius is unity.

RULE II.

From eight times the chord of half the arch, subtract the chord of the whole arch, and one third of the remainder will be the length of the arch *nearly.*

EXAMPLE 2. If the chord, AC or BC, of half the arch be 19.8, and the chord AB of the whole arch 34.4; what is the length of the arch ACB ?



$8 \times 19.8 - 34.4 = 124$; which divided by 3, gives $41\frac{1}{3}$ for the length of the arch.

3. If the chord AB of the whole arch be 6, and the chord AC of half the arch 3.043836, what is the length of the arch ? *Ans.* 6.116896.

4. If the radius of the circle be 9, and the arch contain 38.9424412 degrees, what is its length ? *Ans.* 6.117063.

5. There is an arch of a circle whose chord AB is 50.8 inches, and the chord AC 30.6, what is the length of the arch ? *Ans.* $64\frac{2}{3}$ inches.

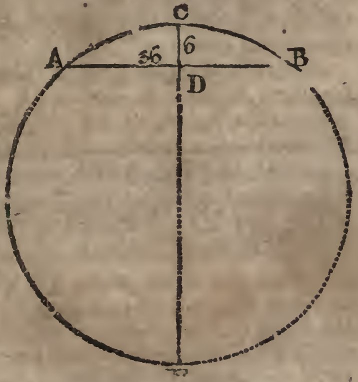
6. The chord AB of the whole arch is 40, and the versed sine DC 15, what is the length of the arch ? *Ans.* $53\frac{1}{3}$.

7. If the radius of the circle be 11.3, and the arch contain 52 degrees 15 minutes, what is its length ? *Ans.* 10.304858, &c.

To find the diameter of a Circle by having the chord and versed sine of the Segment given.

RULE.

Square half the chord; divide it by the versed sine, and to the quotient add the versed sine; the sum will be the diameter.



DEMONSTRATION. The half chord AD, is a mean proportional between the segments, CD, DE, of the diameter CE; that is $AD^2 = CD \times DE$ (by 13.6.) therefore, $\frac{AD^2}{CD} = DE$; to these equals add CD, and $\frac{AD^2}{CD} \times CD = DE \times DC = CE$: which, in words, gives the above rule.

EXAMPLES.

1. Let ACB be a segment given, whose chord AB is 36, and the versed sine CD 6; half 36 is 18, which squared, makes 324; this divided by 6, the quotient is 54: to which add 6, the sum is 60, the diameter of the circle CE.

2. If the chord AB be 18, and versed sine DC 4; what is the diameter EC? *Ans.* $24\frac{1}{4}$.

§ XII. *To find the Area of the Sector of a Circle.*

RULE.

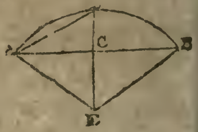
Multiply half the length of the arch by the radius of the circle, and the product is the area of the sector.

EXAMPLES.

1. Let ADBE be the sector of a circle given, whose radius AE or BE is 24.5, and the chord AB 39.2; what is the area?

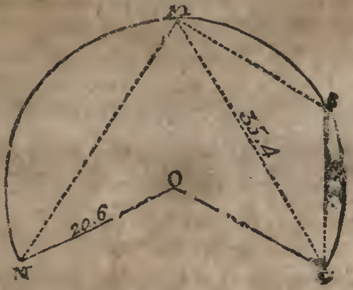
First, find the chord of half the arch, and then the length of the arch by section XI.

From the square of AE 600.25, take the square of AC 384.16, the square root of the remainder, 216.09. gives EC, 14.7; from ED, or its equal AE, take EC, the remainder 9.8 is CD—then to the square of AC 384.16, add the square of CD 96.04, the square root of the sum 480.2 gives AD 21.9135, the chord of half the arch, hence the arch AD is found to be 22.68466; this multiplied by the radius, 24.5, gives 555.77417 for the area of the sector.



2. Let LMNO be a sector greater than a semicircle, whose radius LO or NO, is 20.6, and the chord Nc of half the arch McN 20.35962; also the chord MN of the whole arch McN, that is, of half the arch of the sector, is 35.4; what is the area?

The arch McL will be found, by the following, to be 42.49232, this multiplied by the radius, 20.6, gives 875.341792, the area.



3. Required the area of a sector, less than a semicircle, whose radius is 9, and the chord of the arch 6.

Ans. The length of the arch is 6.116896, and area 27.526032.

4. Required the area of a sector, whose arch contains 18 degrees, the radius being 3 feet.

Ans. The length of the arch is .94247766, and the area 1.41371649.

5. Required the area of a sector, greater than a semicircle, whose radius LO or NO is 10, and the chord MN of half the arch LM c N, *viz.* the chord of the whole arch M c N, is 16.

Ans. As in example 1, you will find Nc=8.9442719, and the length of the arch M c N 18.518050584; hence the whole area will be 185.18050584.

6. What is the area of a sector, less than a semicircle, the chord of the whole arch AB being 50.8, and the chord of half the arch AD 30.6.

Ans. The length of the arch is 64$\frac{2}{3}$; and by taking the square root of the difference of the squares of AD

and AC, DC will be found to be 17.0645; then by the method of finding a diameter, Section XI, as 17.0645 : 25.4 (=AC) :: 25.4 : 37.8071, the remaining part of the diameter; hence the diameter is 54.8716, radius 27.4358, and area of the sector 887.0908.

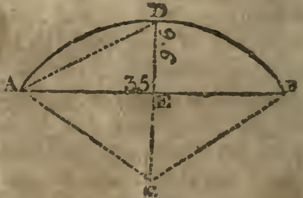
§ XIII. *To find the Area of the Segment of a Circle.*

RULE I.

Find the area of the whole sector CADBC by Section XII. and then (by Section V.) find the area of the triangle ABC, and subtract the area of the triangle from the area of the sector, the remainder will be the area of the segment.—If the segment be greater than a semicircle, add the area of the triangle to the area of the sector, and the sum will be the area of the segment.

EXAMPLES.

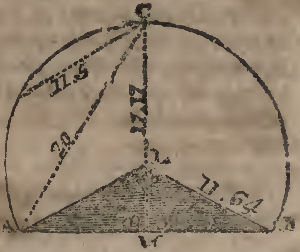
1. Required the area of the segment ADBE whose chord AB is 35, and versed sine DE, 9.6.



First. By the method of finding a diameter, sect. XI. as 9.6 : 17.5 (=AE) :: 17.5 : 31.9, the remaining part of the diameter, which, added to DE, gives 41.5 for the diameter, the half of which, 20.75, is the radius CD; from which take DE, the remainder 11.15, is the perpendicular CE of the triangle ACB.

Second. To the square of AE 306.25 add the square of ED 92.16, the square root of the sum 398.41 is 19.96, the chord AD of half the arch: then, the area of the triangle ABC will be=195.125 the arch AD=20.78; the area of the sector CADB=431.1850; and that of the segment ABD,=236.06.

2. Let MACBM be a segment greater than a semicircle; there are given the base of the segment or chord AB 20.5, the height MC 17.17, the radius of the circle 11.64, the chord of half the arch ACB, *viz.* AC 20, and the chord of one-fourth of the arch 11.5, to find the area of the segment.



Half the arch line will be found, as before, to be=24; which multiplied by the radius 11.64, gives 279.36 for the area of the sector LACB; and half the base AM, multiplied by the perpendicular LM, =5.53 gives 56.6825 for the area of the triangle ABL. Hence 336.0425 is the area of the segment ABC.

RULE II.

To two thirds of the product of the chord and height of the segment, add the cube of the height divided by twice the chord, and the sum will be the area, nearly.

NOTE. When the segment is greater than a semicircle, find the area of the remaining segment, and sub tract it from the area of the whole circle.

EXAMPLES.

1. Required the area of a segment ADBE, less than a semicircle, whose chord AB is 35, and versed sine, or height of the segment, DE 9.6.

$$\tfrac{2}{3} \text{ of } 35 \times 9.6 = 224, \text{ and}$$
$$9.6 \times 9.6 \times 9.6 \div 2 \times 35 = 12.639$$

Area of the segment $= 236.639$

2. The area of a segment greater than a semicircle is required, the chord being 20.5, and height 17.17.

As 17.17 : 10.25 :: 10.25 : 6.119, the height of the remaining segment by sect. XI; hence the diameter is 23.289, and the area of the whole circle 425.983305. By rule 2, the area of the remaining segment is 89.214347, which subtracted from the area of the whole circle, leaves 336.768958 for the area required. See example 2.

3. Required the area of a segment, less than a semicircle, whose chord is 18.9, and height 2.4.

Ans. $\left\{\begin{array}{l}\text{30.601875 by rule 1.}\\ \text{30.6057 by rule 2.}\end{array}\right.$

4. Required the area of a segment, less than a semicircle, whose chord is 23, and height 3.5.

Ans. $\left\{\begin{array}{l}\text{54.5844 by rule 1.}\\ \text{54.598732 by rule 2.}\end{array}\right.$

5. Required the area of a segment, greater than a semicircle, whose chord is 12, and height 18.

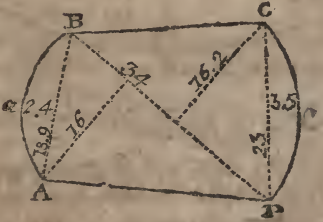
Ans. 297.826 by rule 2. This may also be done by rule 1; for, by what is given, every given line marked in the figure to example 2, may be found.

§ XIV. *To find the Area of Compound Figures.*

Mixed or compound figures are such as are composed of rectilineal and curvilineal figures together.

To find the area of such mixed figures, you must find the area of the several figures of which the whole compound figure is composed, and add all the areas together, and the sum will be the area of the whole compound figure.

Let AaBCcDA be a compound figure, AaB being the arch of a circle whose chord AB is 18.9, and height 2.4; CcD is likewise the arch of a circle whose chord CD is 23, and height 3.5: And ABCD is a trapezium, whose diagonal BD is 34, and the two perpendiculars 16 and 16.2; required the area of this compound figure.

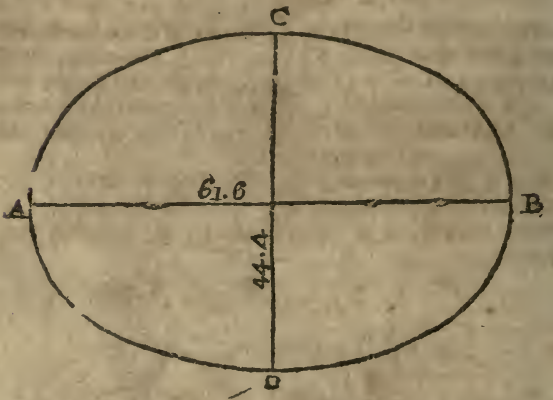


By example 4. section XIII. the area of the segment AaB is - - - - - - - -	30.601875
By example 5. section XIII. the area of the segment CcD is - - - - - - - -	54.5844
Area of the trapezium ABCD - -	547.4
Area of the compound figure - - -	632.586275

§ XV. *To find the* Area *of an* Ellipsis.

RULE.

Multiply the transverse, or longer diameter, by the conjugate or shorter diameter, and that product by .7854; the last product is the area of the ellipsis.



EXAMPLES.

1. Required the area of an ellipsis whose longer diameter AB is 61.6, and shorter diameter CD 44.4.

$61.6 \times 44.4 \times .7854 = 2148.100416$, the area of the ellipsis.

2. The longer diameter of an ellipsis is 50, and the shorter 40, what is the area? *Ans.* 1570.8.

3. The longer diameter of an ellipsis is 70, and the shorter 50, what is the area? *Ans.* 2748.9.

4. Required the area of an ellipsis, whose transverse diameter is 24, and conjugate 18.

Ans. 339.2928.

Demonstration of the Rule.

Put the transverse axis ST$=a$; the conjugate N$n=c$; the height Ta, or Na, of any segment$=x$, and its corresponding ordinate $ab=y$; then by the property of the curve, $y=\frac{c}{a}\sqrt{ax-x^2}$ and the fluxion of the segment bTb, or bN$b=\frac{2c}{a}\times\dot{x}\sqrt{ax-x^2}$. Now, $2\dot{x}\sqrt{ax-x^2}$ is known to be the fluxion of the corresponding circular segment, BTB or bNb. Therefore, if the circular area be denoted be A, the elliptical area will be expressed by $\frac{c}{a}\times$A. Hence it appears that the area of the segment of the ellipsis it to that of the corresponding circle as a to c: and consequently the whole ellipsis, to the whole circle, in the same proportion.

Fig. 1.

Fig. 2.

From a due consideration of the two figures, and the foregoing demonstrations, the following rule, for finding the area of an elliptical segment, is easily deduced.

Given the height of an elliptical segment, and the two diameters, to find the area.

RULE.

1. Subtract the height of the segment, from that diameter of which it is a part; multiply the remainder by the height of the segment, and twice the square root of the product will be the chord of a circular segment, having the same height.

2. Find the area of this segment by rule 2, for circular segments.

3. Then, as that diameter, of which the height of the segment is a part, is to the other diameter; so is the circular area, to the elliptical area.

NOTE. If the chord of the elliptical segment also be given; then, instead of the first part of the above rule, say, as that diameter to which the chord of the segmant is parallel, is to the other diameter, so is the chord of the elliptical segment, to the chord of the circular segment, and proceed as above.

EXAMPLES.

1. In an ellipsis, whose longer axis TS is 120, and shorter N*n* 40, what is the area of a segment thereof, *b*T*b*, cut parallel to the shorter axis N*n*, the height of *a*T being 24?

TS—T*a*=S*a*=96; this multiplied by T*a*, 24, gives 2304; twice the square root of which, is=96, the chord B*a*B, of the circular segment BTB, whose area by the second rule for that purpose, will be found to be 1608. Then, as 120 : 40; or, as 3 to 1 :: 1608 : 536, the area of the elliptic segment *b*T*b*.

NOTE. If the chord *bab* had been given 32; B*a*B might have been found thus, as 40 : 120 :: 32 : 96.

2. In an ellipsis whose longer axis TS is 120, and shorter N*n* 40, what is the area of a segment thereof,

BNB, cut parallel to the longer axis, TS, the height aN being 4?

Proceeding as before, the chord, bab, of the circular segment bNb (*Fig.* 2.) will be found$=24$, and its area $=65\frac{1}{3}$; therefore, as 40 : 120; or as 1:3 :: $65\frac{1}{3}$: 196, the area of the elliptic segment BNB.

3. Required the area of an elliptical segment cut off by a chord or double ordinate, parallel to the shorter diameter, the height of the segment being 10, and the two diameters 35 and 25? *Ans.* 161.878.

4. What is the area of an elliptical segment cut off by a chord, parallel to the longer diameter, the height of the segment being 5, and the two diameters 35 and 25. *Ans.* 97.7083.

5. What is the area of an elliptical segment cut off by a chord, parallel to the shorter diameter, the height being 20, and the two diameters 70 and 50? *Ans.* 647.513997.

Given the two diameters of an Ellipsis to find the circumference.

RULE I.

Multiply the sum of the two diameters by 1.5708, and the product will be the circumference; near enough for most practical purposes.

RULE II.

To half the sum of the two diameters add the square root of half the sum of their squares; multiply this last sum by 1.5708, and the product will be the circumference, extremely near.

EXAMPLES.

1. The two diameters of an ellipsis are 24 and 18, what is the circumference?

By rule 1. $24+18=42$: and $42\times1.5708=65.9736$.

By rule 2. $\left(\dfrac{24+18}{2}+\sqrt{\dfrac{24^2+18^2}{2}}\right)\times1.5708=66.3085$, the circumference of the ellipsis.

2. What is the circumference of an ellipsis whose transverse is 50, and conjugate 40 yards?

Ans. $\begin{cases}141.372 \text{ by rule 1.}\\ 141.8069918 \text{ by rule 2.}\end{cases}$

3. The longer diameter of an ellipsis is 70, and the shorter 50, what is the circumference?

Ans. $\begin{cases}188.496 \text{ by rule 1.}\\ 189.796 \text{ by rule 2.}\end{cases}$

*To find the Area of an Elliptical Ring, or the space included between the Circumferences of two concentric and similar Ellipsis.**

RULE.

Find the area of each ellipsis, and subtract the less from the greater, the remainder will be the area of the ring. Or, from the product of the two diameters of the greater ellipsis, subtract the product of the two diameters of the less; the remainder multiplied by .7854, will be the area of the ring.

Note. This rule will also serve for a circular ring; for when the diameters of each ellipsis become equal, the square of the diameter of the greater circle, diminished by the square of the diameter of the less, and the remainder multiplied by .7854 is the area of the circular ring: Or, multiply the sum of the diame-

* It is here supposed, that the difference between the conjugate diameters is equal to the difference between the transverse diameters; but it is well known, that in this case, the elliptic arches will not be every where equidistant: The difference between the semi-transverse, or semi-conjugate diameters, being the least distance between the arches.

ters by their difference, and that product into .7854 for the area of a circular ring.

EXAMPLES.

1. The transverse diameter AB of an ellipsis is 60, the conjugate CD 47; and the transverse diameter EF of another ellipsis, having the same centre O, is 45, and the conjugate GH 32: required the area of the space contained between their circumferences.

$$AB \times CD = 2820.$$
$$EF \times GH = 1440.$$

Their difference $= 1380.$
Multiplied by $\quad .7854$
Gives the area of the ring $= 1083.852$

2. The ellipsis in *Grosvenor-square* measures 840 links the longest way, and 612 the shortest, within the wall; the wall is 14 inches thick, what quantity of ground does it inclose, and how much does it stand upon?

Ans. { The wall encloses 4 acres 0 roods 60.13 p.
And stands on 195.6103 yards.

3. A gentleman has an elliptical fountain in his garden, whose greatest diameter is 30, and less 24 feet; and has ordered a walk to be paved round it of 5 feet 6 inches in width, with *Purbeck* stone, at 4 shillings per square yard, what will the expence be?

Ans. { The area of the walk is 561.561 feet.
Expence *l.*12 : 9s. : 6d$\frac{3}{4}$. .968.

4. The diameters of two concentric circles are 24 and 18, what is the area of the space included between their circumferences? *Ans.* 197.9208.

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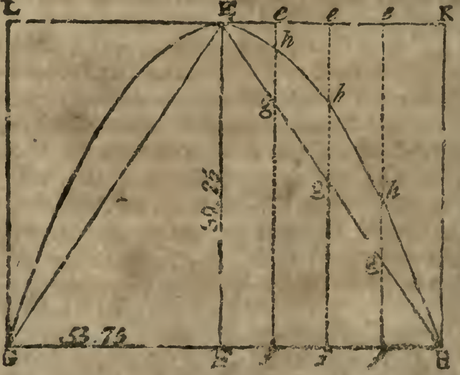
§ XVI. *To find the Area of a Parabola.*

RULE.

Multiply the base or greatest double ordinate, by the perpendicular height, and two thirds of the product will be the area.



EXAMPLES.

1. If the greatest double ordinate GH be 53.75, and the abscissa, or height of the parabola EF, be 39.25, what is the area?

$\frac{2}{3}$ of $53.75 \times 39.25 = 1406.4583$, the area of the parabola.

Let the altitude EF be denoted by x, and the semi-ordinate FH, by y, then if a be the *latus rectum* of the parabola, we have from the nature of the curve, $ax = y^2$ and $x = \frac{y^2}{a}$; hence $\dot{x} = \frac{2y\dot{y}}{a}$, and $y\dot{x} = \frac{2y^2\dot{y}}{a}$ the fluxion of the area HEF; and its fluent $= \frac{2}{3} \times \frac{y^3}{a} = \frac{2}{3} \times xy = \frac{2}{3} \times \text{EF} \times \text{FH}$; therefore the area of the whole parabola GEH, is $= \frac{2}{3} \times \text{GH} \times \text{EF}$, or two thirds of the circumscribing parallelogram.

Note. If the ordinate GH was not at right angles to the axis EF, still the included area would be expressed by two thirds of the base, multiplied into the perpendicular height of the curve above said base or ordinate.

Demonstration of the rule. Let FH, the ordinate, be divided into an infinite number of equal parts n, and suppose lines ef, parallel and equal to EF, to be drawn through each of these equal divisions. Then, by the nature of similar triangles, assuming $\text{EF} = 1$, ge will be equal to $o, \frac{1}{n} + \frac{2}{n} + \frac{3}{n}$, &c.—to—$\frac{n}{n}$; and by a property of the parabola.

(*Emerson's Conics*. B. III. Prop. 7.)

$$\text{EF} = ef : ge :: ge : pe \text{ continually, } viz.$$

$$1 : \frac{1}{n} :: \frac{1}{n} : \frac{1}{n^2}$$

$$1 : \frac{2}{n} :: \frac{2}{n} : \frac{4}{n^2}$$

$$1 : \frac{3}{n} :: \frac{3}{n} : \frac{9}{n^2} \text{ \&c. —— to } \frac{n^2}{n^3}$$

Hence the area of the space EKHhE will be expressed by $o+\frac{1}{n^2}+\frac{4}{n^2}+\frac{9}{n^2}$&c.——to$\frac{n^2}{n^2}$, but the sum of this series (*Emerson's Arithmetic of Infinites, Prop.* 3)$=\frac{1}{3}n$. Now the area of the parallelogram EKHF $=$FH$\times$EF$=n\times1=n$, and $n-\frac{1}{3}n=\frac{2}{3}n$. Hence the area of the semi-parabola is $\frac{2}{3}$ of the area of its circumscribing parallelogram.

2. What is the area of a parabola, whose height or abscissa is 10; and the base, or double ordinate, 16?

Ans. $106\frac{2}{3}$.

3. Required the area of a parabola whose base, or double ordinate, is 38, and the height, or abscissa, 12?

Ans. 304.

CHAPTER II.

The Mensuration of Solids.

SOLID bodies are such as consist of length, breadth, and thickness; as stone, timber, globes, bullets, &c.

A Table of Cubic-Measure.

1728	cubic, or solid inches, make	1	solid	foot.
27	- - - - - feet - -	1	-	yard.
$166\frac{3}{8}$	- - - - - yards - -	1	-	pole.
64000	- - - - - poles - -	1	-	furlong.
512	- - - - - furlongs -	1	-	mile.

The least solid measure is a cubic inch, and all solids are measured by cubes, whose sides are inches, feet, yards, &c.; and the solidity of a body is said

to be so many cubic inches, feet, yards, &c. as will fill the same space as the solid, or as the solid will contain.

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§ I. *Of a Cube.*

A cube is a solid, comprehended under six geometrical squares, and may be represented by a die.

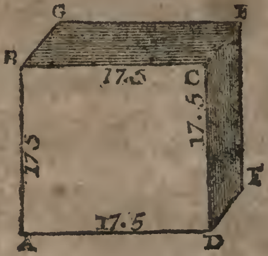
To find the Solidity.

RULE.

Multiply the side of the cube into itself, and that product again by the side; the last product will be the solidity.

EXAMPLES.

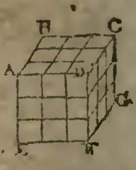
1. Suppose ABCDEFG to be a cubical piece of stone or timber, each side being 17.5 inches, what is the solidity?



$17.5 \times 17.5 \times 17.5 = 5359.375$ the solid content of the cube in inches, which divided by 1728, gives 3.10149 feet.

By Scale and Compasses.

Extend the compasses from 1 to 17.5; that extent turned over twice from 17.5 will reach to 5359, the solid content in inches. Or, extend the compasses from 100 to 175, that extent, turned twice over will reach to 536, which multiplied by 10, gives 5360. Then extend the compasses from 1728 to 1; that extent, turned the same way, from 5359, will reach to 3.1 feet.



Demonstration. If the square ABCD be conceived to be moved down the plane ADEF, always remaining parallel to itself, there will be generated, by such a motion, a solid, having six planes, the two opposite ones of which will be equal and parallel to each other; whence it is called a parallelopipedon, or square prism. And if the plane ADEF be a square equal to the generating plane ABCD, then will the generated solid be a cube. From hence such solids may be conceived to be constituted of an infinite series of equal squares, each equal to the square ABCD; and AE or DF will be the number of terms. Therefore, if the area of ABCD be multiplied into the number of terms AE, the product is the sum of all the terms, and consequently, the solidity of the parallelopipedon or cube. Or, if the base ABCD, be divided into little square areas, and the height AE be divided in a similar manner, you may conceive as many little cubes to be generated in the whole solid, as are equal to the number of the little areas of the base multiplied by the number of divisions which the side AE contains.

From this demonstration it is very plain, that, if you multiply the area of the base of any prism into its length or height, the product will be its solidity.—Any solid figure, whose ends are parallel, equal, and similar, and its sides are parallelograms, is called a *Prism*.

2. What is the solidity of a cube, whose side is 19 feet 4 inches?

Ans. 7226 feet 4 inches 5 parts 4 seconds.

3. A cellar is to be dug, whose length, breadth, and depth, are each 10 feet 4 inches; how many solid feet does it contain, and what will it cost digging, at 1 shilling per solid yard?

Ans. $\begin{cases} \text{The content is } 1103\frac{10}{27} \text{ feet.} \\ \text{Expence } l.2 : 0 : 10\frac{94}{243}. \end{cases}$

4. How many three inch cubes may be cut out of a 12 inch cube? *Ans.* 64.

§ II. *Of a* PARALLELOPIPEDON.

A parallelopipedon is a solid having six rectangular sides, every opposite pair of which are equal and parallel.

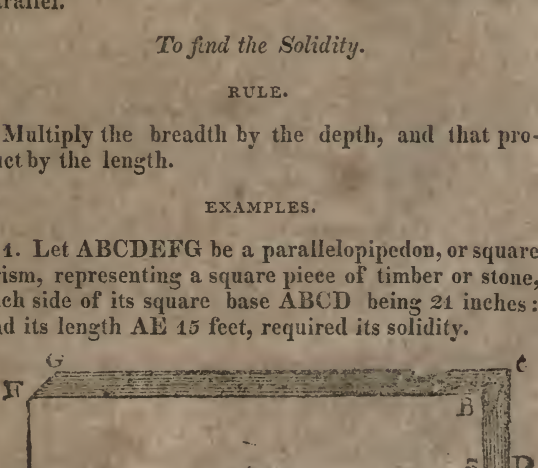
To find the Solidity.

RULE.

Multiply the breadth by the depth, and that product by the length.

EXAMPLES.

1. Let ABCDEFG be a parallelopipedon, or square prism, representing a square piece of timber or stone, each side of its square base ABCD being 21 inches: and its length AE 15 feet, required its solidity.



First, then, multiply 21 by 21, the product is 441, the area of the base in inches; which multiply by 180, the length in inches, and the product is 79380, the solid content in inches. Divide the last product by 1728, and the quotient is 45.9, that is, 45 solid feet and 9 tenths of a foot.

Or thus, by multiplying feet and inches.

Multiply 1 foot 9 inches by 1 foot 9 inches, and the product is 3 feet 0 inches 9 parts; this multiplied again by 15 feet, gives 45 feet 11 inches 3 parts.

By Scale and Compasses.

Extend the compasses from 12 to 21 (the side of the square) and that extent will reach to near 46 feet, being twice turned over from 15 feet; so the solid content is almost 46 feet.

Note. When the breadth and depth of the solid are unequal, a mean proportional between them must be found, by dividing the space between them into two equal parts, for the side of a mean square in inches; then proceed as above. For, as 12 is to the side of a mean square in inches, so is the length in feet to a fourth number; and so is this fourth number to the content in foot measure, of any parallelopipedon.

2. If a piece of timber be 25 inches broad, 9 inches deep, and 25 feet long, how many solid feet are contained therein? *Ans.* 39 feet 0 inches 9 parts.

3. If a piece of timber be 15 inches square at the end, and 18 feet long, how many solid feet are contained in it; and suppose I want to cut off a solid foot from one end of it, at what distance from the end must I cut it?

Ans. { The solidity is 28.125 feet.. And 7.68 inches in length will make 1 foot.

4. If a piece of squared timber be 2 feet 9 inches long, 1 foot 7 inches broad, and 16 feet 9 inches long,

how many feet of timber are in that piece; and how much in length will make a solid foot?

Ans. { The solid content is 72.93 feet, or 72 feet 11 inches 2 parts 3 seconds; and 2.756 inches in length make 1 solid foot.

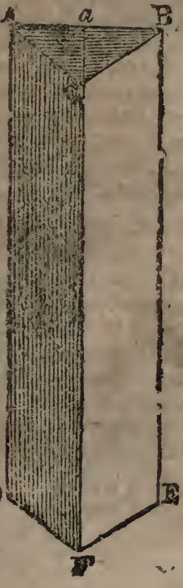
§ III. *Of a Prism.*

A prism is a solid contained under several planes, and having its bases similar, equal, and parallel.

To find the Solidity.

RULE.

Multiply the area of its base or end, by the height or length, and the product will be the solidity.



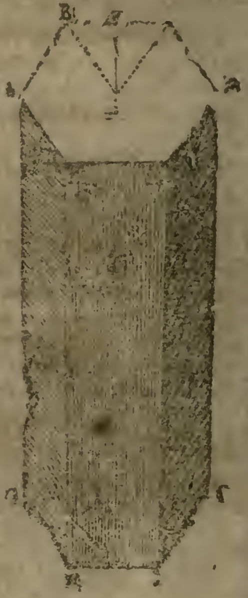
EXAMPLES.

1. Let ABCDEF be a triangular prism, each side of the base being 15.6 inches, the perpendicular *Ca* 13.51 inches, and the length of the solid 19.5 feet. How many solid feet are contained therein?

Multiply the perpendicular of the triangle 13.51 by half the side 7.8, and the product is 105.378, the area of the base; which multiply by the length 19.5, and the product is 2054.871; which divide by 144, and the quotient is 14.27 feet *fere*, the solid content.

By Scale and Compasses.

First, find a mean proportional between the perpendicular and half side, by dividing the space upon the line, between 13.51 and 7.8 into two equal parts; so shall you find the middle point between them to be at 10.26, which is the mean proportional sought: by this means the triangular solid is brought to a square one, each side being 10.26 inches. Then extend the compasses from 12 to 0.26; that extent, turned twice downwards from 19.5 feet, the length, will at last fall upon 14.27, which is 14 feet and a little above a quarter.



2. Let ABCDEFGHIK represent a prism, whose base is a hexagon, each side being 16 inches, the perpendicular from the centre of the base to the middle of one of the sides (ab) 13.84 inches, and the length of the prism 15 feet: the solid content is required.

Multiply half the sum of the sides 48 by 13.84, and the product is 664.32, the area of the hexagonal base (by § VIII. p. 96.) which multiplied by 15 feet, the length, the product is 9964.8; which divided by 144, the quotient will be 69.2 feet, the solid content required.

By Scale and Compasses.

First, find a mean proportional between the perpendicular, and half the sum of the sides; that is divide the space between 13.84 and 48, and the middle point will be 25.77, the side of a square equal in area to the base of the solid. Then extend the compasses from 12 to 25.77; that extent will reach (being twice turned over) from 15 feet, the length, to 69.2 feet, the content.

To find the Superficial Content of any Prism.

RULE.

Multiply the circumference of the base by the length of the prism; the product will be the upright surface; to which add the area of the bases; the sum will be the whole superficial content.

EXAMPLES.

In the hexagonal prism last mentioned, the sum of the sides, or circumference of the end, being 96, and the length 15 feet, that is, 180 inches: which multiplied by 96, the product is 17280 square inches; to which add twice 664.32, the areas of the two bases, the sum is 18608.64, the area of the whole, which is 129.226 feet.

1. Each side of the base of a triangular prism is [illegible] inches, and the length 12 feet 5 inches; what is the solidity and surface?

Ans. { The solidity 10.79 feet
{ Surface 54.5089 feet.

2. A hexagonal prism measures 28 inches across the centre of the end, from corner to corner, and is 134 inches in length; required the solidity and surface.

Ans. { The solidity is 39.48835 feet. Surface 85.2392 feet.

3. A decagonal prism, or pillar, measures 50 inches in circumference, and is 30 feet high; required the solidity and surface.

Ans. { The solidity 40.074 feet. Surface 127.6716 feet.

4. The gallery of a church is supported by 10 octagonal prisms of wood, which measure each 48 inches in circumference, and are each 12 feet high; what will be the expence of painting them at 10 pence per square yard? *Ans.* *l.*2 : 7s : $9\frac{1}{3}$.

5. A trapezoidal prism of earth, or part of a canal, is to be dug, whose perpendicular depth is 10 yards, the width at the top 20 yards, at the bottom 16, and the length 50 yards, the two ends being cut perpendicularly down; how many solid yards of earth are contained in this part? *Ans.* 9000 yards.

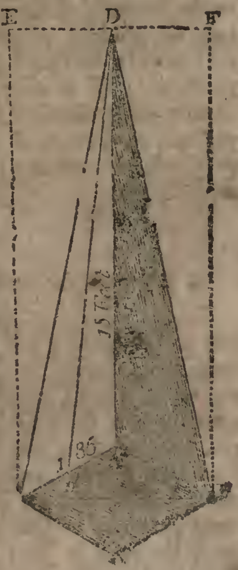
§ IV. *Of a* PYRAMID.

A pyramid is a solid figure, the base of which is a polygon, and its sides plain triangles, their several vertical angles meeting together in one point.

To find the Solidity.

RULE.

Multiply the area of the base by a third part of the altitude, or length; and the product is the solid content of the pyramid.



EXAMPLE 1. Let ABD be a square pyramid, having each side of its base 18.5 inches, and the perpendicular height CD 15 feet, what is the solidity?

Multiply 18.5 by 18.5, and the product is 342,25, the area of the base, in inches, which multiply by 5, a third part of the height, and the product is 1711.25; this divided by 144 gives 11.88 feet, the solid content.

To find the Superficial Content

RULE.

Multiply the slant height by half the circumference of the base, and the product will be the upright surface.—To which the area of the base may be added, for the whole surface.

Note. This rule will serve for the surface of all pyramids. Perhaps it may be proper here to acquaint the learner, that the slant height of any pyramid is not the height from one of the corners of the base to the vertex or top, but from the middle of one side of the base. And the perpendicular height of a pyramid, is a line drawn from the vertex, to the middle or centre of the base; hence it will be necessary to find the distance between the centre of the base of a pyramid, and the middle of one of the sides.—This distance may always be found by multiplying the tabular perpendicular in section V. p. 84, by one of the sides of the base; then, if to the square of this number, you add the square of the perpendicular height of the pyramid, the square root of the sum will give the slant height.

EXAMPLES.

1. Required the surface of the foregoing pyramid.

To the square of the perpendicular height D*e* 15 feet or 180 inches, add the square of *de* 9.25, the distance from the centre *e*, of the base, to the middle *d* of one of the sides; which, in a square base, is always equal to half the side. The square root of the sum, *viz.* 32485.5625 is 180.24 nearly, the slant height D*d*. Now half the circumference of the base is 37, which multiplied by 180.24, gives 6668.88 inches for the upright surface; to which add 342.25 the area of the base, the sum is 7011.13 inches, the whole surface equal to 48.69 feet nearly.

Demonstration of the rule. Every pyramid is a third part of a prism, that has the same base and height (by *Euclid*, XII. 7.)

That is, the solid content of the pyramid ABD (in the last figure) is one third part of its circumscribing prism ABEF. Now the solidity of a prism is found

by multiplying the area of the base into the height; therefore the solidity of a pyramid will be found by multiplying the area of the base by the height, and taking one third of the product. You may very easily prove a triangular pyramid to be a third part of a prism of equal base and altitude, mechanically, by making a triangular prism of cork, and then cutting that prism into three Pyramids, in a diagonal direction.

2. Let ABCD be a triangular pyramid, each side of the base being 21.5 inches, and its perpendicular height 16 feet; required the solidity and surface.

First, the area of the base, by sect. V. is 200.19896 inches, which multiplied by 64, the third part of the height in inches, gives 12812.73344 inches, the solidity, equal to 7.41477 feet. Again, the distance from the centre of the triangle ABC to the middle of one of the sides, as *d* in the side AC, is 6.20652; to the square of this number, which is 38.5208905104, add the square of the height 36864 inches, the square root of the sum 36902.5208905104, is 192.10028 inches, the slant height *d*D. Hence the upright surface is 6195.23403 inches, and the whole surface 6395.43299 inches, or 44.41272 feet.

3. Let ABCDEFGH be a pyramid, whose base is a heptagon, each side of it being 15 inches, and the perpendicular height of the pyramid, HI, 13.5 feet; required the solidity and superficies.

First, 15 multiplied by 1.0382617, the tabular perpendicular for a heptagon (sect. VIII.) gives 15.5739255 inches, the distance from the centre I to the middle of the side AB; this multiplied by 52.5, half the sum of the sides of the base, gives 817.63108875 inches, the area of the base. This last number multiplied by 4.5, one third of the height, and divided by 144, will give 25.55 feet, the solidity.—Again, if to the square of 15.5739255, you add the square of the height in inches, the square root of the sum 26486.54715548 will be 162.74688, the slant height of the pyramid; this multiplied by 52.5 gives 8544.2112 inches, the upright surface, to which add the area of the base, and the whole surface will be 9361.84228875 inches, or 65.0128 feet.

By Scale and Compasses.

First, find a mean proportional between 15.57 and 52.5, by dividing the space between them into two equal parts, and you will find the middle point to be 28.6, the side of a square equal in area to the base of the pyramid; then extend the compasses from 12 to 28.6, that extent will reach from 4.5 (twice turned over) to 25.55 feet, the solid content. *Or*, extend the compasses from 1 to the area of the base, that extent

will reach from one-third of the height to the solidity; the dimensions being all of the same name.

4. There is a triangular pyramid, the sides of its base are 13, 14, and 15 feet, and its altitude 63 feet, what is the solidity? *Ans.* 1764 feet.

5. Each side of the base of a hexagonal pyramid is 10, and the perpendicular height 45, what is the solidity and superficies?

Ans. { Solidity 3897.1143 feet.
Superficies 1634.580328 feet.

6. Required the solidity and surface of a square pyramid, each side of the base being 3 feet, and the perpendicular height 24 feet.

Ans. { Solidity 72 feet.
Surface 153.2808 feet.

7. A square pyramidal stone, whose slant height is 21 feet, and each side of its square base 30 inches, is to be sold at 7s. per solid foot; and the polishing of the upright surface will cost 8d. per foot; what will be the expence of the stone when finished?

Ans. l.18 : 15s. : $8\frac{1}{4}$d.

8. The spire of a church is an octagonal pyramid (built of stone) each side of the base being 5 feet 10 inches, and its perpendicular height 45 feet; also each side of the cavity, or hollow part, at the base is 4 feet 11 inches, and its perpendicular height 41 feet; I desire to know how many solid yards of stone the spire contains?

Ans. { Solidity of the whole 2464.509614 feet.
The cavity 1595.180393 feet.
And the stone-work 32.19738 yards.

§ V. *Of a* Cylinder.

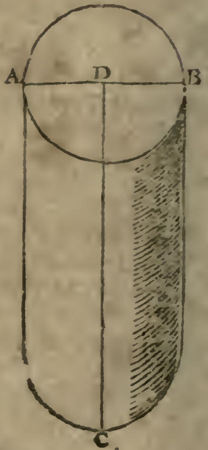
A cylinder is a solid, having its bases circular, equal, and parallel, in form of a rolling stone.

To find the Solidity.

RULE.

Multiply the area of the base by the length, and the product is the solid content.

EXAMPLES.



1. Let ABC be a cylinder, whose diameter AB is 21.5 inches, and the length CD is 16 feet, the solid content is required.

First, square the diameter 21.5, and it makes 462.25; which multiply by .7854, and the product is 363.05115; then multiply this by 16, and the product is 5808.8184. Divide this last product by 144, and the quotient is 40.339 feet, the solid content.

By Scale and Compasses.

Extend the compasses from 13.54 to 21.5, the diameter, that extent (turned twice over from 16, the length) will at last fall upon 40.34, the solid content.

Note. 13.54 is the diameter of a circle whose area is 144 inches; and as 13.54 is to the diameter of any cylinder's base in inches, so is the length of that cylinder, in feet, to a fourth number; and so is this fourth number, to the solid content of the cylinder in feet.

When the circumference is given in inches, and the length in feet, extend from 42.54 to the circumference of the cylinder, that extent will reach from the length,

turned twice over, to the content in feet. For, as 42.54 is to the circumference of any cylinder, in inches, so is the length of that cylinder in feet to a fourth number; and so is this fourth number to the solidity of the cylinder in feet.

42.54 is the circumference of a circle whose area is 144.

To find the Superficial Content.

First, (by chap. I. sect. IX. prob. 1,) find the circumference of the base 67.54, which multiplied by 16, the product is 1080.64; and this divided by 12, the quotient is 90.05 feet, the upright surface; to which add 5.04 feet, the sum of the areas of the two bases, and the sum is 95.09 feet, the whole superficial content.

2. If a piece of timber be 96 inches in circumference, and 18 feet long, how many feet of timber are contained in it, supposing it to be perfectly cylindrical? *Ans.* 91.676 feet.

3. If a piece of timber, perfectly cylindrical, be 86 inches in circumference, and 20 feet long; how many solid feet are contained therein? *Ans.* 81.74634 feet.

4. The diameter of the base of a cylinder is 20.75 inches, and its length 4 feet 7 inches; what is its solidity and surface? *Ans.* { Solidity 10.7633 feet. Surface 29.595 feet.

5. I have a rolling stone, 44 inches in circumference, and am to cut off 3 cubic feet from one end; whereabouts must the section be made?

Ans. At 33.64 inches.

6. A person bespoke an iron roller for a garden, the outside diameter to be 20 inches, the thickness of the metal $1\frac{1}{2}$ inch, and length of the roller 50 inches; now supposing every cubic inch to weigh $4\frac{1}{4}$ ounces, what will the whole come to at $3\frac{1}{4}$d. per pound?

Ans. { Solidity 4358.97 inches. Weight 1157.8514 pounds. Cost *l.* 15 : 13s. : 7d.

§ VI. *Of a Cone.*

A cone is a solid, having a circular base, and growing proportionably smaller, till it ends in a point, called the vertex, and may be nearly represented by a sugar loaf.

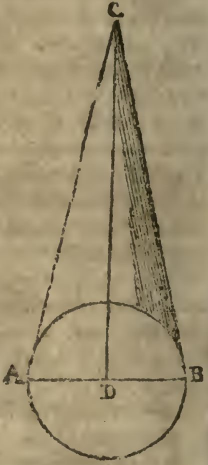
To find the Solidity.

RULE.

Multiply the area of the base by a third part of the perpendicular height, and the product is the solid content.

EXAMPLES.

1. Let ABC be a cone, the diameter of whose base AB is 26.5 inches, and the height DC 16.5 feet: required the solidity.



First, square the diameter 26.5, and it is 702.25; this multiplied by .7854 gives 551.54715 inches, the area of the base; which multiplied by 5.5, one third of the height, gives 3033.509325; this divided by 144 gives 21.066 feet, the solid content.

By Scale and Compasses.

Extend the compasses from 13.54 to 26.5, the diameter, that e ent turned twice over from 5.5 (a third part of the height,) it will at last fall upon 21.06 feet, the content.—See the remark on the number 13.54, in section V.

To find the Superficial Content.

The rule given in sect. IV. for finding the superficies of a pyramid, will serve to find the superficies of a cone.

First, find the slant height thus: To the square of the radius, 13.5, which is 175.5625, add the square of the height in inches, 39204, the square root of the sum, 39379.5625, is 198.4428, the slant height. The radius of the base, 13.25, multiplied by 3.1416 gives half the circumference of the base 41.6262, and this product multiplied by the slant height, gives 8260.41968136 for the upright, or convex surface; to which add the area of the base 551.54715, and the sum is 8811.96683136 inches, the whole surface, equal to 61.194 feet.

Demomstration. Every cone is the third part of a cylinder of equal base and altitude (by *Euclid*, XII. 10.) The truth of this may easily be conceived, by only considering, that a cone is but a round pyramid; and therefore it must needs have the same ratio to its circumscribing cylinder, as a multangular pyramid hath to its circumscribing prism, *viz.* as 1 to 3. For, the base of the multangular pyramid may consist of such a number of sides, that the difference between its circumference, and that of the circle, will be less than any assignable magnitude.

The curve superficies of every right cone, is equal to half the rectangle of the circumference of its base into the length of its side.

For the curve surface of every right cone is equal to the sector of a circle, whose arch BC is equal to the periphery of the base of the cone, and radius AB equal to the slant side of the cone: which

will appear very evident, if you cut a piece of paper in the form of a sector of a circle, as ABC, and bend the sides AB and AC together, till they meet, and you will find it to form a right cone.

I have omitted the demonstrations of the superficies of all the foregoing solids, because I thought it needless, they being all composed of squares, parallelograms, triangles, &c. which figures are all demonstrated before. And if the area of all such figures as compose the surface of the solid, be found separately, and added together, the sum will be the superficial content of the solid.

2. The diameter of the base of a cone is 10, and its perpendicular height 68.1; required the solidity and superficies. *Ans.* { Solidity 1782.858. Superficies 1151.134076.

3. What is the solidity of an elliptical cone, the greater diameter of its base being 15.2, the less 10, and the perpendicular height 22? *Ans.* 875.4592.

4. What is the solidity and superficies of a cone, whose perpendicular height is 10.5 feet, and the circumference of its base 9 feet?

Ans. { Solidity 22.56093 feet. Superficies 54.1336 feet.

5. What is the solidity and superficies of a cone, whose slant height is 32 feet, and the circumference of its base 24 feet?

Ans. { Solidity 485.4433316 feet. Surface 429.83804 feet.

6. What will the painting of a conical church spire come to at 8d. per yard; supposing the circumference of the base to be 64 feet, and the perpendicular height 118 feet?

Ans. { Superficies 3789.76 feet. Expence *l.*14 : 0s. : $8\frac{3}{4}$d.

§ VII. *Of the Frustum of a Pyramid.*

A frustum of a pyramid is the remaining part, when the top is cut off by a plane parallel to the base.

To find the Solidity.

GENERAL RULE.

Multiply the areas of the two bases together, and to the square root of the product add the two areas; that sum, multiplied by one third of the height, gives the solidity of any frustum.

RULE II.

If the Bases be Squares.

To the rectangle (or product) of the sides of the two bases add the sum of their squares; that sum, being multiplied into one-third part the frustum's height, will give its solidity.

RULE III.

If the Bases be Circles.

To the product of the diameters of the two bases add the sum of their squares; this sum, multiplied by the height, and then by .2618, or one-third of .7854, the last product will be the solidity.

RULE IV.

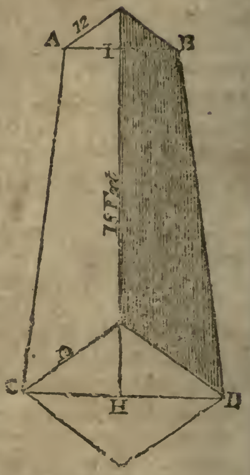
If the Bases be regular Polygons.

Add the square of a side of each end of the frustum, and the product of those sides into one sum; multiply this sum by one-third of the tabular area be-

longing to the polygon (sect. VIII. p. 99.) and this product by the height, for the solidity.

EXAMPLES.

1. Let ABCD be the frustum of a square pyramid, each side of the greater base 18 inches, each side of the less 12 inches, and the height 18 feet; the solidity is required.



BY THE GENERAL RULE.

The square of 18 is 324, and the square of 12 is 144; also the product of these two squares is 46656, the square root of which is 216, a mean area.

Area of the greater base - - - - -	324
Area of the less base	144
Mean area - - -	216
Sum -	684

which multiplied by 6, one third of the height, gives 4104; this divided by 144 gives 28.5 feet the solidity.

BY RULE 2.

Product of the sides of the bases 18 and 12 is	216
Square of the greater side 18 is - - - - - -	324
Square of the less side 12 is - - - - - - -	144
Sum - -	684

Then proceed as above.

To find the Superficial Content.

RULE.

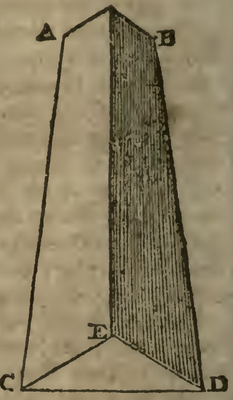
Multiply half the sum of the perimeters of the two bases by the slant height, and to that product add the areas of the two bases for the whole superficies.

NOTE. The slant height of any frustum, whose ends are regular polygons, is a line drawn from the middle of one side of the less end, to the middle of its parallel side at the greater end. And, the perpendicular height, is a line drawn from the centre of the less end, to the centre of the greater; or it is a perpendicular let fall from the middle of one of the sides of the less end, upon the surface of the greater end. Hence the slant height, and perpendicular height, will be two sides of a right angled triangle; the base of which will be equal to the difference between the radii of the inscribed circles of the two ends of the frustum. And this base may always be found, by multiplying the difference between a side at each end of the frustum, by the tabular perpendicular in section VIII. p. 100. The perpendicular height of the pyramid of which any frustum is a part, may readily be found, by saying, as the base found above, is to the perpendicular height of the frustum; so is the radius of the inscribed circle of the frustum's base, to the perpendicular height of the pyramid. The radius of the inscribed circle is found by multiplying a side of the base, by the tabular perpendicular, section VIII.

EXAMPLE. Required the superficies of the foregoing frustum.

The perimeter of the greater base	72 inches.
The perimeter of the less base is	48 inches.
Sum - -	120
Half sum - -	60

From 18, a side of the greater end, take 12, a side of the less; the difference 6, multiplied by the tabular perpendicular .5, gives 3. The perpendicular height of the frustum in inches is 216: to the square of 216, which is 46656, add the square of 3, which is 9, the square root of the sum 46656 is 216.02083 inches, the slant height, which multiplied by 60 gives 12961.2498 inches. To this product add the areas of the two ends 324, and 144, and the whole surface will be 13429.2498 inches, or 93.2586 feet.



Example 2. Let ABCD be the frustum of a triangular pyramid, each side of the greater base 25 inches, each side of the less base 9 inches, and the length 15 feet; the solid content of it is required.

BY RULE 4.

The square of 25 is	625
The square of 9 is	81
Product of 25 and 9 is	225
Sum	931

The tabular area is .433013, one-third of which is .1443377, this multiplied by 931, and then by 15, produces 2015.6759805, which divided by 144, the quotient is 13.9977 feet, the solidity.

OR THUS, BY THE GENERAL RULE.

The square of 25, multiplied by the tabular area, (section VIII.) gives the area of the greater base 270.633125: in a similar manner the area of the less base will be found to be 35.07405; the square root of the product of these two areas 9492.200569805625 is 97.427925.

Area of the greater base - - -	270.633125
Area of the less base - - -	35.074053
Mean proportional between them -	27.427925
Sum - -	403.135103

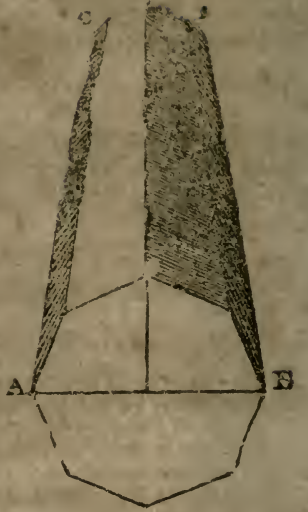
which multiplied by 5, one third of the height, and divided by 144, gives 13.9977 feet, the solidity as before.

To find the Superficial Content.

The perimeter of the greater base is	75 inches.
The perimeter of the less base is -	27 inches.
Sum -	102
Half sum -	51

From 25, a side of the greater end, take 9, a side of the less, the difference 16, multiplied by the tabular perpendicular .2886751, gives 4.6188016.—The perpendicular height of the frustum in inches, is 180; to the square of 180, which is 32400, add the square of 4.6188016, which is 21.33332822, is 180.0592 inches, the slant height; which multiplied by 51 gives 9183.0192 inches. To this product add the areas of the two ends 270.633125, and 35.074053, and the whole surface will be 9488.726378 inches, or 65.893933 feet.

EXAMPLE 3. Suppose ABCD to be the frustum of a pyramid, having an octagonal base, each side of it being 9 inches, each side of the less base 5 inches, and the length, or height, 10.5 feet; the solidity is required.



BY RULE 4.

The square of 9 is	81
The square of 5 is	25
Product of 5 and 9 is - -	45
Sum - -	151

Tabular area is 4.828427, one third of which is 1.6094756; this multiplied by 151, and then by 10.5, produces 2551.8235638, which divided by 1.44, the quotient, is 17.721 feet, the solidity.

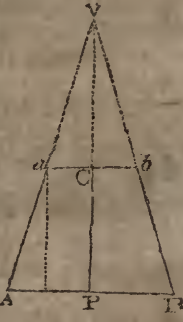
To find the Superficial Content.

The perimeter of the greater base is	72 inches.
The perimeter of the less base is	40
Sum -	112
Half sum -	56

From 9, a side of the greater base or end, take 5, a side of the less, the remainder 4 multiplied by the tabular perpendicular 1.2071068, gives 4.8284272. The perpendicular height of the frustum in inches is 126; to the square of 126, which is 15876, add the square of

4.8284272, which is 23.3137092257, the square root of the sum 15899.3137092257, is 126.09248, the slant height; which multiplied by 56 gives 7061.17888 inches. To this product add the areas of the two ends, 391.102587, and 120.710675 (found by rule 2, section VIII.) and the whole surface will be 7572.992142 inches, or 52.590223 feet.

DEMONSTRATION. From the rules delivered in the IVth and VIth sections, the preceding rules may easily be demonstrated.



Suppose a pyramid ABV, to be cut by a plane at *ab*, parallel to its base AB, and it were required to find the solidity of the frustum, or part A*ab*B.

Let D=AB, a line, or diameter of the greater base.
d=*ab*, a similar line of the less base.
h=PC, the height of the frustum.
A=area of the greater base.
a=area of the less.

AP—aC : PC : : aC : CV, by similar triangles;

$$\text{viz.}\ \frac{D}{} - \frac{d}{2} : h :: \frac{d}{2} : \frac{hd}{D-d} = CV.$$

$$\text{but } PC + CV = h + \frac{hd}{D-d} = \frac{hD}{D-d} = PV$$

$$\frac{hd}{D-d} \times \frac{a}{3} = \text{solidity of the pyramid } abV$$

$$\frac{h-D}{D-d} \times \frac{A}{3} = \text{solidity of the pyramid ABV.}$$

$$\text{Hence } \frac{hD}{D-d} \times \frac{A}{3} - \frac{hd}{D-d} \times \frac{a}{3} = \frac{AD-ad.}{D-d} \times \frac{h}{3} \text{ — the solidi-}$$

ty of the frustum ABab. Now all similar figures are to each other as the squares of their homologous sides. (*Euclid* VI, and 19th and 20th; also XII. and 2d.) Therefore $A : a :: D^2 : d^2$ or $\frac{A}{D^2}=\frac{a}{d^2}$ put each of these equal to m; then $A=mD^2$ and $a=md^2$, and multiplying these quantities together, we get $aA=m^2D^2de$, consequently $\sqrt{aA}=mDd$. For A and a substitute their values in the expression for the frustum's solidity, and it becomes $\frac{mD^3-md^3}{D-d}\times\frac{h}{3}=\frac{D^3-d^3}{D-d}\times\frac{h}{3}\times m=D^2+Dd+d^2$; $\times\frac{h}{3}\times m=mD^2+mDd+md^2\times\frac{h}{3}$; restore the values, of A and a, &c. and the rule becomes $A+\sqrt{aA}+a$; $\times\frac{h}{3}$ which is the *general rule.*

Corollary 1. If the bases be squares, of which D and d are each a side, then D^2+Dd+d^2; $\times\frac{h}{3}$ is the solidity and is the second rule.

Cor. 2. If the bases be circles, of which D and d are diameters then D^2+Dd+d^2; $\times h\times .2618$ is the solidity, which is the third rule.

Cor. 3. If the bases be regular polygons, and t represent the tabular number in section VIII. also D and d represent a side of each base, the rule becomes $\overline{D^2}\times t+\sqrt{\overline{D^2t\times d^2t}}+d^2t$; $\times\frac{h}{3}=D^2+Dd+d^2$; $\times\frac{t}{3}\times h$, the solidity, which is rule 4.

That the rule for finding the superficies of a frustum is true, will readily appear, when we consider that if the ends be regular polygons, the sides will consist of as many trapeziods as there are sides in the polygon; and the common height of these trapeziods will be the slant height of the frustum; and the sum of their parallel sides, the sum of the perimeters of the ends.

But when the bases are circles, the rule is not so obvious. It is shewn in section VI. that the curve superficies of a right cone is equal to the rectangle of half the circumference of the base, into the length of the slant side; *viz.* that it is equal to the area of the sector of a circle, whose arch is equal to the circumference of the base, and radius equal to the slant side of the cone.

Let $DE=P$, the circumference of the great base, and $BC=p$, the circumference of the less base of the frustum; AD the slant height$=S$, now, because similar arches of circles are as their radii.

$$DE : BC :: AD : AB$$

viz. $$P : p :: S : \frac{Sp}{P}$$

Hence $$BD=AD-AB=S-\frac{Sp}{P}=\frac{P-p}{P}\times S$$

The area of the sector $ADE=\frac{PS}{2}$, and

The area of sector $ABC=\frac{P}{2}\times\frac{Sp}{P}=\frac{Sp^2}{2P}$

Also $ADE-ABC=$ curve surface of the frustum.

$$BDEC=\frac{PS}{2}-\frac{Sp^2}{2P}=\frac{PS^2-p^2S}{2P}=\frac{P^2-p^2}{2P}\times S=\frac{P+p}{2};$$

$\times\frac{P-p}{P}\times S$, which is the rule.

Note. From the foregoing demonstration we have the following rule for finding the area of a segment of a sector B D E C, or the front of a circular arch, built with stones of equal length.

RULE.

Multiply half the sum of the bounding arches (DE and BC) by their distance (BD), and the product will give the area.

Example 4. If each side of the greater end of a piece of squared timber be 25 inches, each side of the less end 9 inches, and the length 20 feet; how many solid feet are contained in it?

Ans. By rule 2, 43.1018 feet.

5. If a piece of timber be 32 inches broad, and 20 inches deep, at the greater end; and 10 inches broad, and 6 inches deep, at the less end; how many solid feet are contained therein, the length being 18 feet?

Ans. By the general rule, 37.3316 feet.

6. A portico is supported by four pillars of marble, each having eight equal sides, whose breadth at the greater end is 7.5 inches, and at the less end $4\frac{1}{4}$ inches, and their length 12 feet 9 inches; I desire to know how many solid feet they contain, and what they will come to at 12s. 10d. *per* solid foot?

Ans. { Solidity 60.52927823 feet.
Expence *l.* 38 : 16s. : $9\frac{1}{2}$d.

7. In a frustum of a square pyramid, each side of the greater end is 5 feet, each side of the less end 3 feet, and the height 8 feet; required the height and solidity of that pyramid of which this frustum is a part?

Ans. { The height is 20 feet, see p. 143.
The solidity $166\frac{2}{3}$ feet.

8. If each side of the greater base of the frustum of a hexagonal pyramid be 13, each side of the less base 8, and the length 24, what is the solidity and superficies?

Ans. { Solidity 7004.412896.
Superficies 2141.7642,

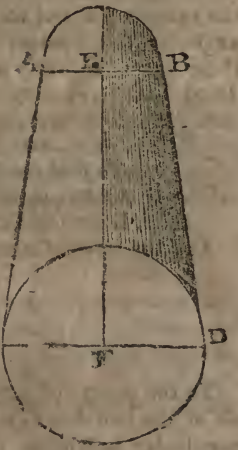
§ VIII. *Of the Frustum of a* CONE.

A frustum of a cone, is that part which remains when the top end is cut off by a plane parallel to the base.

The solidity may be found either by the general rule, or rule 3, of the preceding section. The superficies may also be found by the rule given in that section.

EXAMPLES.

1. Let ABCD be the frustum of a cone, whose greater diameter CD is 18 inches, the less diameter AB 9 inches, and the length 14.25 feet; the solid content is required.



BY THE GENERAL RULE.

The square of 18 is 324, which multiplied by .7854 gives 254.4696 inches, area of the greater base. The square of 9 is 81, which multiplied by .7854 gives 63.6174, area of the less base.

The square root of the product of these areas 16188.69433104 is 127.2348.

Area of the greater base - -	254.4696
Area of the less base, - - -	63.6174
Mean proportional - - - -	127.2348
	Sum 445.3218

This multiplied by 4.75, one third of the height, gives 2115.27855, which divide by 144, and the quotient is 14.68943 feet, the solidity.

Note. The diameter of the less base being exactly half that of the greater, in this example, the operation might have been performed shorter: for the area of the less base in such cases is one fourth of that of the greater, and the mean area double that of the less base, or half that of the greater.

OR THUS, BY RULE 3.

The square of 18 (the greater diameter) is 324, and the square of 9 (the less diameter) is 81, and the rectangle, or the product of 18 by 9, is 162, the sum of these three is 567, which multiplied by the height 14.25 feet, gives 8079075; which multiplied by .2618, and divided by 144, gives 14.68943 feet as before.

To find the Superficial Content.

The circumference of the greater base, by chap. I. section IX. is	56.5488
And the circumference of the less base is	28.2744
Sum	84.8232
Half sum	42.4116

To the square of 4.5, the difference between the radii of the two bases, add the square of 171, the perpendicular height in inches; the square root of the sum 29261.27, is 171.0592 inches, the slant height; which multiplied by 42.4116 produces 7254.89436672, to which add the sum of the areas of the two ends, and the whole superficies will be 7572.98136672 inches, or 52.59 feet.

2. If a piece of timber be 9 inches in diameter at the less end, 36 inches, at the greater end, and 24 feet long; how many solid feet are contained therein?

Ans. 74.2203 feet.

3. If a piece of timber be 136 inches in circumfer-

ence at one end, 32 inches at the other, and 21 feet long; how many solid feet are contained therein?

Ans. 95.34816 feet.

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§ IX. *Of a Wedge.*

A wedge is a solid, having a right angled parallelogram for its base, and two of its sides meeting in an edge.

To find the Solidity.

RULE.

To twice the length of the base add the length of the edge, multiply the sum by the breadth of the base, and that product by the perpendicular height of the wedge; and one-sixth of the last product will be the solidity.

EXAMPLES.

1. The perpendicular height OI, of a wedge is 24.8 inches; the length CK of the edge, 110 inches; the length AE of the base 70, and its breadth AB 30 inches; what is the solidity?

70 length of the base AE.
2
———
140
110 length of the edge CK.
———
250 sum.
30 breadth of the base.
———
7500 product.
———

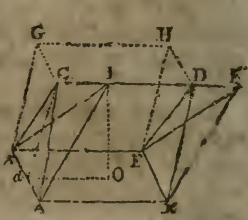
7500
24.8 perp. OI.
———
60000
30000
15000
———
6)186000.0
———
31000 inches; which divided by 1728 gives 17.9398 feet, the solidity.

2. Required the solidity of a wedge, whose altitude is 14 inches, its edge 21 inches, and the length and breadth of its base 32 and 4.5 inches?

Ans. $892\frac{1}{2}$ inches.

3. Required the solidity of a wedge, whose altitude is 2 feet 4 inches, length of the edge 3 feet 6 inches, length of the base 5 feet 4 inches, and breadth 9 inches? *Ans.* 4.131944, &c. feet.

DEMONSTRATION OF THE RULE.



Let ABCDEF represent a wedge: now when the length of the edge CD is equal to the length AE of the base ABFE, the wedge is evidently equal to half a parallelopipedon ABGCHDEF, having the same base and altitude as the wedge; and this will always be true, whether the end ABC of the wedge be perpendicular to its base ABFE, or inclined as ABI, since parallelopipedons standing on the same base and between the same parallels are equal to each other (*Euclid*, XI. and 29.) But when the edge CD of the wedge is longer or shorter than the base, by any quantity DK or CI, it is evident that the wedge will be greater or less than the half parallelopipedon aforesaid, by a pyramid whose base is EFD, or ABC, and perpendicular altitude DK or CI. Let the length AE of the base of the wedge be represented by L, and the breadth AB by B; call the length of the edge l, and the perpendicular height AC, aC, or OI, h. Then, the solidity of the wedge will be expressed by $\frac{BL}{2}\times h$ or $\frac{Bh}{2}\times L=\frac{BLh}{2}$, when the edge is of the same length

as the base: The solidity of the pyramid ABCI will be expressed by $\frac{Bh}{2} \times \frac{L-l}{3}$ when the length ID of the edge is less than that of the base; and the solidity of the pyramid EFDK will be expressed by $\frac{Bh}{2} \times \frac{l-L}{3}$, when the length of the edge GK is greater than that of the base. Hence, $\frac{BLh}{2}; + \frac{Bh}{2} \times \frac{l-L}{3} = \frac{2L+l}{6} \times Bh$, the rule, when the edge is longer than the base, and $\frac{BLh}{2}; \frac{Bh}{2} \times \frac{L-l}{3} = \frac{2L+l}{6} \times Bh$, the rule when the edge is shorter than the base.

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§ X. *Of a* PRISMOID.

A prismoid is a solid somewhat resembling a prism; its bases are right angled parallelograms, and parallel to each other, though not similar, and its sides four plain trapezoidal surfaces.

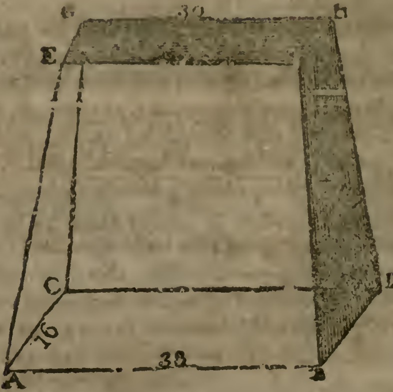
To find the Solidity.

RULE.

To the greater length add half the less length, multiply the sum by the breadth of the greater base, and reserve the product.

Then, to the less length, add half the greater length, multiply the sum by the breadth of the less base, and add this product to the other product reserved; multi-

ply the sum by a third part of the height, and the product is the solid content.



EXAMPLES.

1. Let ABCDEFGH be a prismoid given, the length of the greater base AB 38 inches, and its breadth AC 16 inches; and the length of the less base EF 30 inches, and its breadth 12 inches, and the height 6 feet; the solid content is required.

To the greater length AB 38, add half EF the less length 15, the sum is 53; which multiplied by 16, the greater breadth, the product is 848; which reserve.

Again, to EF 30, add half AB 19, and the sum is 49; which multiplied by 12 (the less breadth EG) the product is 588; to which add 848 (the reserved product,) and the sum is 1436; which multiply by 2, (a third part of the height,) and the product is 2872; divide this product by 144, and the quotient is 19.94 feet, the solid content.

2. One end of a prismoid is a square, each side of which is 13; the other a right angled parallelogram,

whose length is 12 and breadth 5; what is the solidity, the perpendicular height being 20? *Ans.* $2263\frac{1}{3}$.

3. In the neighbourhood of Newcastle, and in the county of Durham, &c. the coals are carried from the mines in a kind of waggon, in the form of a prismoid. The length at the top is generally about 6 feet $9\frac{1}{2}$ inches, and breadth 4 feet 7 inches; at the bottom the length is 3 feet 5 inches, and breadth 2 feet $5\frac{1}{2}$ inches; and the perpendicular depth 3 feet $11\frac{1}{4}$ inches. Required the solidity.

Ans. 126340.59375 cubic inches, or 73.11376 feet.

4. How many gallons of water, reckoning 282 cubic inches to a gallon, are contained in a canal 304 feet by 20 at top, 300 feet by 16 at bottom, and 5 feet deep? *Ans.* 166590.6383 gallons.

DEMONSTRATION OF THE RULE.

The prismoid is evidently compassed of two wedges whose bases are equal to the bases of the prismoid, and their height equal to the height of the prismoid. Let L equal the length of the greater base AB, which of course will be the length of the edge of the less wedge; l equal the length of the less base EF, or the edge of the greater wedge: B the breadth of the greater base AC, b the breadth of the less base EG, and h

the common height. Then the solidity of the wedge whose base is ABDC will be expressed by $\frac{2L+l}{6}\times Bh$, and the solidity of the wedge whose base is EFHG by $\frac{2L+l}{6}\times bh$; hence the solidity of the whole prismoid will be $\frac{2l+L}{6}\times Bh, \times \frac{2l+L}{6}\times bh = \overline{\frac{L+l}{2}\times B; +l+\frac{L}{2}\times b;}\times\frac{h}{3}$, which is the rule exactly.

This rule is demonstrated section II. Prob. XIV. of Emerson's Fluxions, and in a similar manner at page 179, second edition, of Simpson's Fluxions; also in Holiday's Fluxions, page 302.

NOTE. If $\frac{L+l}{2}=M$, and $\frac{B+b}{2}=m$, the above rule becomes $BL+bl+4Mm; \times\frac{h}{6}$, *that is,* To the sum of the areas of the two ends add four times the area of a section parallel to, and equally distant from, both ends; this last sum multiplied into one-sixth of the height, will give the solidity.

It is shewn in proposition III. page 456, part IV. of Dr. Hutton's mensuration, *quarto edition,* that this last rule is true, for all frustum's whatever, and for all solids whose parallel sections are similar figures. And, Mr. Moss, in his Guaging, page 175, third edition,

says, that it is *nearly* true, let the form of the solid be what it will.

The following rule for measuring a *cylindroid* given by Mr. Hawney, from Mr. Everard's Guaging, has been by some teachers considered as erroneous, others again have imagined it to be true. The general rule, which has just been described, will most certainly give the content either *exactly*, or as near as possible, provided the figure of the middle section between CD and AB (see the following figure) can be accurately determined. Half the sum of CD and AB, in the following example, is 35, and half the sum of AB and EF is 20, the two diameters of the middle section; therefore it must either be an ellipsis, or a curve of the oval kind, whose area, perhaps, cannot be easily determined. If the section be an ellipsis (and it cannot materially differ from one) the rule given by Mr. Hawney exactly agrees with the above general rule. But Mr. Moss, in prob. VI. sect. VIII. of his Guaging, has shewn that the figure of the middle section between CD and AB, can *never* be an ellipsis, unless the parallel ends AB and CD are similar ellipsis, and similarly situated; *viz.* the transverse and conjugate diameters of each end, respectively parallel to each other, and this circumstance can never happen but when the solid is the frustum of an elliptical cone. The rule therefore cannot be strictly true, though sufficiently near for any practical purpose.

To measure a Cylindroid*; or, the Frustum of an elliptical Cone.*

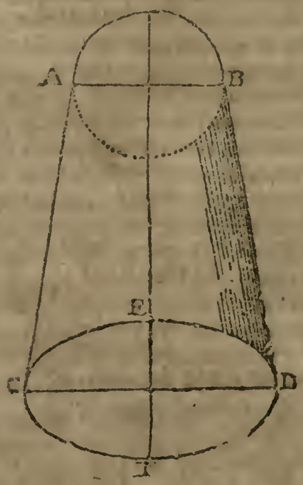
RULE.

To the longer diameter of the greater base, add half the longer diameter of the less base, multiply the

sum by the shorter diameter of the greater base, and reserve the product.

Then, to the longer diameter of the less base, add half the longer diameter of the greater base; and multiply the sum by the shorter diameter of the less base; add this product to the former reserved product, and multiply the sum by .2618 (on -third of .7854) and then by the height, the last product will be the solidity.

EXAMPLES.



1. Let ABCD be a cylindroid, whose bottom base is an ellipsis, the transverse diameter being 44 inches; and the conjugate diameter 14 inches; and the upper base is a circle, of which the diameter is 26 inches;

and the height of the frustum is 9 feet; the solidity is required.

44=CD	26=AB
13=half AB	22=half CD
57 sum	48 sum
14=EF	26=AB
228	288
57	96
798 product reserved.	1248 product.
	798 add reserved product.
	2046 sum.
	2046
	.2618
	16368
	2046
	12276
	4092
	535.6428
	9=height.

4820.7852 which, divided by 144, gives 33.4776 feet the solidity.

2. The transverse diameter of the greater base of a cylindroid is 13, conjugate 8; the transverse diameter of the less base 10, and its conjugate 5.2. If the length of the cylindroid be 20, what is the solidity?

Ans. 1203.2328.

3. I desire to know what quantity of water an elliptical bath will hold, the longer diameter at the top be-

ing 12 feet, and shorter 7; the longer diameter at the bottom 10 feet, and the shorter 6 feet; and the depth 4 feet; reckoning 282 cubic inches to a gallon?

Ans. 1379.6303 gallons.

§ XI. *Of a Sphere or Globe.*

A sphere, or globe, is a round solid body, every part of its surface being equally distant from a point within, called its centre; and it may be conceived to be formed by the revolution of a semicircle round its diameter.

To find the Solidity.

RULE I.

Multiply the cube of the diameter by .5236, and the product will be the solidity.

RULE II.

Multiply the diameter of the sphere into its circumference, and the product will be the superficies; which multiplied by one-third of the radius, or a sixth part of the diameter, will give the solidity.

EXAMPLES.

1. Required the solidity of a globe, ABCD, whose diameter AB or CD is 20 inches.

BY RULE 1.

$20\times 20\times 20\times .5236 \doteq 4188.8$, the solidity.

BY RULE 2.

$20\times 3.1416=62.832$, the circumference.
$20\times 62.832=1256.64$, the superficies.
$20\times 1256.64\div 6=4188.8$, the solidity, in inches, $=2.424$ feet.

2. The diameter of the earth is 7970 miles, what is its surface in square miles, and solidity in cubic miles?

Ans. { Surface 199557259.44 miles.
Solidity 265078559622.8 miles.

3. The circumference of a sphere is 1, what is its solidity and superficies?

Ans. { Solidity .0168868.
Superficies .3183099.

4. What is the solid and superficial content of a globe, whose diameter is 30?

Ans. { Solidity 14137.2.
Superficies 2827.44.

5. The circumference of a globe is 50.3, what is its solidity and superficies?

Ans. { Solidity 2149.073728.
Superficies 805.3526.

6. A globe, a cube, a cylinder,
All three in surface equal are,* *viz.* 3.1416
In solidity what do they differ? *each.*

Ans. The solidity of the globe is .5237, the cube .378877, and cylinder .4275176; so that of all solids, under the same superficies, the globe is the greatest.

* *First, for the globe.* If the superficies of a globe be divided by 3.1416, the quotient will be the square of the diameter, the square root of which will be the diameter.—Hence the solidity is easily found.

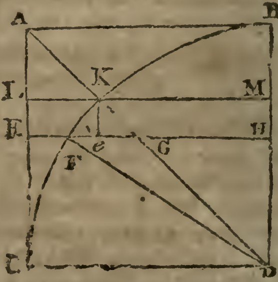
Second, for the cube. The cube has 6 equal surfaces, therefore the whole surface divided by 6, will give the square of the side of the cube; the square root will be the side.—Hence the solidity is found.

Third, for the cylinder. The diameter and depth of the cylinder are here supposed equal.—The superficies of a cylinder, whose diameter and depth are each an unit, will be found to be 4.7124; and the superficies of similar cylinders are to each other as the squares of their diameters, therefore 4.7124 is to the square of 1, as the superficies of the cylinder is to the square of its diameter; the square root of which will be the diameter, and likewise the depth.—Hence the solidity of the cylinder is found.

DEMONSTRATION OF THE RULE, &c.

That every Sphere is two-thirds of its Circumscribing Cylinder, may be thus proved.

Let the square ABCD, the quardrant CBD, and the right angled triangle ABD, be supposed all three to revolve round the line BD as an axis: Then will the square generate a cylinder, the quadrant a hemisphere, and the triangle a cone, all of the same base and altitude.



By *Euclid*, I. and 47, $FD^2 = FH^2 + DH^2$, but $FD = EH$, and $GH = DH$ because EH is parallel to CD, therefore $\overline{EH}^2 = \overline{FH}^2 + \overline{GH}^2$; and as circles are to each other as the squares of their diameters, or radii (*Euclid*, XII. and 2,) it follows that the circle described by EH, is equal to the two circles described by FH

and GH; take away the circle described by FH, and there remains the circle described by EH—circle described by FH (*viz.* the annulus, or ring, described by EF between the sphere and cylinder)=circle described by GH. And it is evident, that this property will hold in every section similar to EH; but the cone, hemisphere, and cylinder, may be conceived to be made up of an infinite number of such sections, therefore when the hemisphere is taken out of the cylinder, the remaining part is equal to the solidity of the cone; but the cone is one-third of the cylinder, therefore the hemisphere must be two-thirds, and what is here proved with respect to the halves of the proposed solids, holds equally true with the wholes. *Therefore every sphere is two-thirds of its circumscribing cylinder.* Now if D be the diameter and height of a cylinder, its solidity will be $D^2 \times .7854 \times D = D^3 \times .7854$, hence the solidity of its inscribed sphere will be $D^3 \times .7854 \times \frac{2}{3} = D^3 \times .5236$, which is the first rule.

The Surface of a Sphere is equal to the Curve Surface of its Circumscribing Cylinder.

Take FK, an extremely small part of the quadrantal arch CB, and suppose lines LM and EH to be drawn through K and F parallel to each other. Then, because FK is extremely small, it may be considered as a straight line, and the angle FKD as a right angle. The figure FKMH, by revolving round MH, will form the frustum of a cone, whose slant side is FK; and the figure ELMH, by revolving in the same manner, will form a cylinder, whose perpendicular height is LE. But the surface of the frustum is equal to FK×half the sum of the circumferences of two circles whose radii are KM and FH; or these being extremely near to each other, the surface of the frustum will be equal to FK×circumference of a circle, whose radius is KM, or FH. Let C=the circum-

ference of a circle whose radius is CD or EH, and c=the circumference of a circle whose radius is FH or KM: then the surface formed by the revolution of FKMH about MH, is equal to $c \times FK$; and the cylindrical surface formed by the revolution of ELMH, is equal to $C \times LE$. But the triangles FeK and DMK are equiangular and similar; for FKD and eKM are right angles; take away the common angle eKG, and there remains FKe=DKM; and the angles FeK and KMD are right angles; therefore

DK or CD : KM : : KF : Ke or LE.

But the circumferences of circles are to each other as their radii, therefore

$C : c :: $ CD or LM : KM, consequently.

$C : c :: $ KF : Ke or LE.

Hence $C \times LE = c \times KF$. That is the cylindrical surface formed by the revolution of ELMH, is equal to the spherical surface formed by the revolution of FKMH. And, if more parallel planes be drawn, extremely near to each other, the small parts of the cylindrical surface will be equal to the correspondent parts of the spherical surface; and therefore the sum of all the parts of the cylindrical surface, will be equal to the sum of all the parts of the spherical surface; that is, the surface of the half cylinder will be equal to the surface of its inscribed hemisphere, and the surface of the whole cylinder equal to the surface of its inscribed sphere. Hence if D=the diameter as before, then

$D \times 3.1416 \times D$; $\times \frac{D}{6}$ is the *second* rule,= $\overline{D^3} \times .5236$, the same as the *first* rule.

Corollary 1. The surface of the sphere is $\frac{2}{3}$ of the whole surface of its circumscribing cylinder.

For $D^2 \times 3.1416$ is the surface of the sphere=*curve* surface of the cylinder; and the area of the two ends

of the cylinder$=D^2\times.7854$;$+D^2\times.7854=1.5708\,D^2$, therefore the surface of the whole cylinder is$=3.1416\,D^2+1.5708\,D^2=4.7124\,D^2$; two-thirds of which$=3.1416\,D^2$, the surface of the sphere.

Cor. 2. Hence the surface of the whole sphere is equal to the area of four great circles of the same sphere; or to the rectangle of the circumference and diameter.

Cor. 3. The surface of any segment or zone of a sphere, is equal to the curve surface of a cylinder, of the same height; and whose diameter is equal to that of the sphere.

For the zone formed by the revolution of FKMH about MH, is equal to the surface of the cylinder formed by the revolution of ELMH.

Investigation of General Rules for finding the Solidity of any Segment, or Zone of a Sphere.

Let ACy represent a triangular pyramid, whose side Ay is infinitely small. The sphere may be considered to be constituted of an infinite number of such pyramids, yCe, eCa, &c. whose bases compose the spherical surface, and altitudes are equal to the radius of the sphere, their common vertex being the centre C. And, consequently, the sphere, or any sector thereof, is equal to a pyramid, the area of whose base is equal to the spherical surface, and height equal to the radius of the sphere. Let bT the height of the segment aTm be called h, the diameter of the sphere D, then will its circumference be 3.1416 D; hence by Cor. 3, preceding, the surface of the spherical segment aTm will be expressed by 3.1416 Dh. And the solidity of the spher-

ical sector $CaTm$ (supposing CM to be drawn) by what has just been observed, will be $3.1416\ Dh \times \frac{D}{6} = .5236\ D^2h$.

By the property of the circle $\overline{D-h}\times h=\overline{ab}^2$; but $\overline{ab}^2\times 3.1416\times\frac{bC}{3}=\overline{D-h}\times h\times 3.1416\times\frac{\overline{D-2h}}{6}=\overline{D-h}\times h \times .5236\times\overline{D-2h}=D^2h-3Dh^2+2h^3;\times .5236$, the solidity of the cone Cam. Now, the solidity of the cone taken from the solidity of the sector, leaves the solidity of the segment, *viz.*

$D^2h\times .5236;-D^2h+3Dh^2-2h^3;\times .5236=$

$3Dh^2-2h^3;\times .5236=\overline{3D-2h}\times h^2\times .5236$, which is *one* rule for the solidity of a segment; and, it is evident from the property of the circle, that this rule will be true, whether the segment be greater or less than the hemisphere, or whatever be the magnitude of h, provided it be not greater than D. Or, if $r=ab$ the radius of the segment's base, then $\overline{D-h}\times h=r^2$, therefore $D=\frac{r^2\times h^2}{h}$, for D in the above rule, substitute its value, and the rule becomes $\overline{\frac{3r^2\times 3h^2}{h}-2h};\times h^2\times .5236=\overline{3r^2+h^2}\times h\times .5236$, an useful rule, when the diameter of the sphere is not given.

To find the Solidity of a Zone of a Sphere.

Call the height of the greater segment H, and the height of the less h; also R the radius of the greater base, and r that of the less: then it is evident the difference between the solidity of these two segments will be the solidity of the zone. Hence $\overline{3R^2+H^2}\times H$

$\times .5236; \overline{3r^2+h^2}\times h\times .5236=(3R^2H+H^3;-\overline{3r^2h+h^3})$ $\times .5236$. Put $a=H-h$ the breadth of the zone, and D the diameter of the sphere: then, by the property of the circle, $\overline{D-H}\times H=R^2$, and $D-h\times h=r^2$ from the former of these $D=\frac{R^2\times H^2}{H}$, and from the latter $D=\frac{r^2+h^2}{h}$; therefore $\frac{R^2+H^2}{H}=\frac{r^2+h^2}{h}$, and from the above $a=H-h$. Exterminate the values of H and h, and the above Theorem $(\overline{3R^2H+H^3;-3r^2h+h^3})$: $\times .5236$, will become $R^2+r^2+\frac{a^2}{3};\times a\times 1.5708$.

If one of the radii pass through the centre, as in the zone A G m a, then $R^2=\frac{D^2}{4}=\overline{ab}^2+\overline{bC}^2=r^2+a^2$; hence the last theorem becomes $r^2+\frac{2}{3}a^2;\times a\times 3.1416$ $=\frac{1}{4}D^2-\frac{1}{3}a^2;\times a\times 3.1416$.

Hence $r^2+\frac{2}{3}a^2:\times a\times 6.2832=\frac{1}{4}D^2-\frac{1}{3}a^2\times a\times 6.2832$ *will express the solidity of the middle zone* amKN, being double of the former, where a is half the altitude, and r=half the diameter of each end. Put A for the whole altitude, and $d=2r$ the diameter of each end, and the theorems become $d^2+\frac{2}{3}A^2;\times A\times .7854$ $=D^2-\frac{1}{3}A^2;\times A\times .7854$.

To find the Solidity of the Segment of a Sphere.

RULE I.

From three times the diameter of the sphere subtract twice the height of the segment; multiply the remainder by the square of the height, and that product by .5236, and the last product will be the solidity.

R

RULE II.

To three times the square of the radius of the segment's base, add the square of its height; multiply this sum by the height, and the product by .5236, and the last product will be the solidity.

EXAMPLES.

1. Let ABCD be the frustum of a sphere; suppose AB, the diameter of the frustum's base to be 16 inches, and CD, the height, 4 inches; the solidity is required?

BY RULE I.

$AD^2 \div DC, + DC = CE$; that is, $8 \times 8 \div 4, + 4 = 20$ the diameter; therefore, $(20 \times 3 - 4 \times 2) \times 4 \times 4 \times .5236 = 435.6352$, the solidity of the frustum.

BY RULE 2.

$(3AD^2 + CD^2) \times CD \times .5236 = 435.6352$, the solidity, as before.

2. What is the solidity of a segment of a sphere, whose height is 9, and the diameter of its base 20?
Ans. 1795.4244.

3. What is the solidity of the segment of a sphere, whose diameter is 20 feet, and the height of the segment 5 feet? *Ans.* 654.5 feet.

4. The diameter of a sphere is 21, what is the solidity of a segment thereof, whose height is 4.5?
Ans. 572.5566.

5. The diameter of the base of a segment of a sphere is 28, and the height of the segment 6.5; required the solidity? *Ans.* 2144.99285.

To find the Solidity of the Frustum, or Zone, of a Sphere.

GENERAL RULE.

Add together the square of the radii of the ends, and one-third of the square of their distance, *viz.* the height; multiply the sum by the height, and that product by 1.5708, the last product will be the solidity.

Or, for the middle Zone of a Sphere.

1 To the square of the diameter of the end, add two-thirds of the square of the height; multiply this sum by the height, and then by .7854, for the solidity.

2 Or, from the square of the diameter of the sphere, subtract one-third of the square of the height of the middle zone; multiply the remainder by the height, and then by .7854, for the solidity.

EXAMPLES.

1. Required the solidity of the zone of a sphere AGKN (see the figure, p. 179,) whose greater diameter AG is 20 inches, less diameter NK 15 inches, and distance between the ends, or height CB 10 inches.

$(AC^2+NB^2+\frac{1}{3}CB^2)\times CB\times 1.5708=2977.9744764$, the solidity.

2. Required the solidity of the middle zone $a\,m$ KN of a sphere, whose diameter AG is 22 inches; the top and bottom diameters $a\,m$ and NK of the zone being each 16.971 inches, and the height Bb 14 inches?

Ans. 4603.4912 inches.

3. Required the solidity of a zone whose greater diameter is 12, less diameter 10, and height 2?

Ans. 195.8264.

4. Required the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 feet, and the breadth of the zone 4 feet?

Ans. 61.7848 feet.

5. Required the solidity of the middle zone of a sphere, whose diameter is 5 feet, and the height of the zone 4 feet? *Ans.* 61.7848 feet.

To find the Convex Surface of any Segment, or Zone, of a Sphere.

RULE.

Multiply the circumference of the whole sphere by the height of the segment, or zone, and the product will be the convex surface.

EXAMPLES.

1. If the diameter of the earth be 7970 miles, the height of the frigid zone will be 252.361283 miles; required its surface?

Ans. 6318761.107182216 miles.

2. If the diameter of the earth be 7970 miles, the height of the temperate zone will be 2143.6235535 miles; required its surface?

Ans. 53673229.812734532 miles.

3. If the diameter of the earth be 7970 miles, the height of the torrid zone will be 3178.030327 miles; required its surface?

Ans. 79573277.600166504.

NOTE. The surfaces of the two frigid zones, by example 1, are - - - - - -	6318761.107182216 6318761.107182216
The surfaces of the two temperate zones, by example 2, are - - - - - - - - -	53673229.812734532 53673229.812734532
The torrid zone, example 3, is - - - - - - - -	79573277.600166504
Surface of the whole globe, agreeing exactly with example 2, p. 175. - - -	199557259.440000000

§ XII. *Of a Spheroid.*

A spheroid is a solid formed by the revolution of an ellipsis about its axis. If the revolution be made about the longer axis, the solid is called an *oblong*, or *prolate* spheroid, and resembles an egg: but if the revolution be made round the shorter axis, the solid is called an *oblate* spheroid, and resembles a turnip, or an orange. The earth and all the planets are considered as oblate spheroids. In an oblong or prolate spheroid, the shorter axis is called the revolving axis, and the longer axis the fixed axis: but in an oblate spheroid, the longer axis is called the revolving axis, and the shorter axis is called the fixed axis.

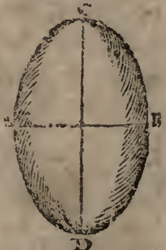
To find the Solidity.

RULE.

Multiply the square of the revolving axis by the fixed axis, and that product by .5236 for the solidity.

EXAMPLES.

1. Let ADBC be a *prolate* spheroid, whose longer axis CD is 55 inches, and shorter axis AB 33; required the solidity?

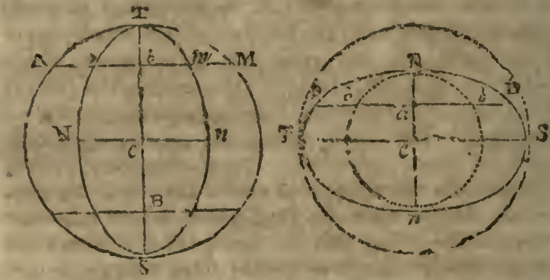


$AB^2 \times CD \times .5236 = 31361.022$, the solid content, required.

2. What is the solidity of an *oblate* spheroid, whose longer axis is 55 inches, and shorter axis 33 inches?
Ans. 52268.37 inches.

3. The axes of an oblong or *prolate* spheroid are 50 and 30; required the solidity? *Ans.* 23562.

4. The axes of an *oblate* spheroid are 50 and 30; required the solidity? *Ans.* 39270.



Demonstration. Suppose the figure $NTnSN$ to represent a spheroid, formed by the rotation of the semi-ellipsis TNS, about its transverse axis TS.

Let $D=TS$, the length of the spheroid, and the axis of the circumscribing sphere; and $d=Nn$, the diameter of the greatest circle of the spheroid:

Then $\overline{TS}^2 : \overline{Nn}^2 :: \overline{Ab}^2 : \overline{ab}^2$, by section XV. p. 127, that is, $D^2 : d^2 :: \overline{Ab}^2 : \overline{ab}^2$; but all circles are to each other as the squares of their radii. Now, the solidity of the sphere may be imagined to be constituted of an infinite numb r of circles, whose radii are parallel to Ab; and the spheroid of an infinite number of circles whose radii are parallel to ab. Therefore $D^2 : d^2$ :: solidity of the sphere whose diameter is D : Solidity of the spheroid whose revolving axis is d;

viz. $D^2 : d^2 :: D^3 \times .5236 : \dfrac{5236\ D^3\ d^2}{D^2} = .5236\ Dd^2$.

Again for the *oblate* spheroid, by *Emerson's* Conics, Book I. prob. 7, $\overline{NC}^2 : \overline{TC}^2 :: na \times aN : \overline{Ba}^2$; but by

the property of the circle $na \times aN = \overline{ba}^2$,

therefore $\overline{NC}^2 : \overline{TC}^2 :: \overline{ba}^2 : \overline{Ba}^2$, *viz.*

$d^2 : D^2 :: \overline{ba}^2 : \overline{Ba}^2$; hence from the same manner of reasoning as above, $d^2 : D^2$ :: solidity of the sphere, whose diameter is d : the solidity of a spheroid, whose revolving axis is D.

Hence $d^2 : D^2 :: d^3 \times .5236 : \dfrac{.5236 d^3 D^2}{d^2} = .5236 d D^2$.

Now from these proportions, between the sphere and its inscribed or circumscribed spheroid, it will be very easy to deduce theorems for finding the solid content, either of the segment or middle zone of any spheroid, having the same height with that of the sphere; for,

As the solidity of the whole sphere is to the solidity of the whole spheroid, so is any part of the sphere to the like part of the spheroid.

1st. *To find the solidity of a segment of a spheroid.*

$\overline{TS}^2 : \overline{Nn}^2 :: \overline{Ab}^2 : \overline{ab}^2$ in the prolate spheroid.

$\overline{Nn}^2 : \overline{TS}^2 :: \overline{ab}^2 : \overline{aB}^2$ in the oblate spheroid.

Hence $\overline{TS}^2 : \overline{Nn}^2$ :: solidity ATM : solidity aTm,

$\overline{Nn}^2 : \overline{TS}^2$ :: solidity bNb : solidity BNB.

But the solidity of ATM or bNb may be found by rule the first, section XI. p. 181.

2d. *To find the solidity of the middle zone of a spheroid.*

Let f=TS the fixed axis. } for the oblong
r=Nn the revolving axis } spheroid.
h=Bb the height of the middle zone or frustum.
D=AM the diameter of one end of the spherical zone.
d=am the corresponding diameter of the spheroidal zone.

By section XI. the solidity of the middle zone of the sphere is $D^2+\frac{2h^2}{3}\times h\times .7854=3D^2+2h^2;\times h\times .2618$; but $\overline{Ab}^2=\overline{AC}^2-\overline{Cb}^2$ *viz.* $\frac{D^2}{4}=\frac{f^2}{4}-\frac{h^2}{4}$; therefore $D^2=f^2-h^2$, hence the solidity of the spherical zone$=3f^2-h^2;\times h\times .2618$. But $.5236\,f^3 : .5236\,fr^2 :: 3f^2-h;\times h\times .2618$: the solidity of the spheroidal frustum, *viz.*

$f^2 : r :: 3f^2-h^2;\times h\times .2618$: the solidity of the spheroidal frustum. By the property of the ellipsis $\overline{TS}^2 : \overline{Nn}^2 :: \overline{AM}^2 : \overline{am}^2$, that is,

$$f^2 : r^2 :: f^2-h^2 : d^2, \text{ hence } f^2=\frac{r^2h^2}{r^2-d^2};$$

Consequent. $\frac{r^2h^2}{r^2-d^2} : r^2 :: \frac{3r^2h}{r^2-d^2}-h^2;\times h\times .2618$: the solidity of the spheroidal frustum$=2r^2+d^2;\times h\times .2618$. The rule for the oblate spheroidal zone will be exactly the same, putting r for TS and d for BaB.

To find the Solidity of the Segment of a Spheroid; the Base of the Segment being parallel to the revolving Axis of the spheriod.

RULE.

From three times the fixed axis subtract twice the height of the segment, multiply the remainder by the square of the height, and that product by .5236.— Then as the square of the fixed axis, is to the square of the revolving axis; so is the last product, found above, to the solidity of the spheroidal segment.

EXAMPLES.

1. In a *prolate* spheroid, the transverse or fixed axis TS is 100, the conjugate or revolving axis Nn is 60, and the height bT of the segment aTm is 10: required the solidity?

(3TS—2Tb) $\times Tb^2 \times .5236 = 14660.8$; then as $TS^2 : Nn^2 :: 14660.8 : 5277.888$, the solidity of the segment aTm.

2. In an *oblate* spheroid, the transverse or revolving axis TS is 100, the conjugate or fixed axis Nn is 60, and the height Na of the segment BNB is 10: required the solidity?

(3Nn—2Na) $\times Na^2 \times .5236 = 8377.6$; then, as $Nn^2 : TS^2 :: 8377.6 : 23271\frac{1}{9}$, the solidity of the segment BNB.

Now, square of $Nn = 3600$ and square of $TS = 10000$. As $3600 : 10000 :: 8377.6 : 23271\frac{1}{9}$ solidity of the spheroidal segment BNB.

3. The axes of a prolate spheroid are 50 and 30; what is the solidity of a segment whose height is 5, and its base parallel to the conjugate or revolving axis? *Ans.* 659.736.

4. The axes of an oblate spheroid are 50 and 30; what is the solidity of a segment whose height is 6, and its base parallel to the transverse or revolving axis? *Ans.* 4084.08.

5. If the axes of a prolate spheroid be 10 and 6, required the area of a segment whose height is 1, and its base parallel to the conjugate or revolving axis? *Ans.* 5.277888.

6. The axes of a prolate spheroid are $36\frac{2}{3}$ and 18; what is the solidity of a segment whose height is $6\frac{1}{2}$, $\times$ 6 and its base parallel to the conjugate or revolving axis? *Ans.* 517.1306.

To find the Solidity of the middle Zone of a Spheroid, having circular ends; the middle Diameter, and that of either of the ends being given.

RULE.

To twice the square of the middle diameter, add the square of the diameter of the end; multiply the sum by the length of the zone, and the product again by .2618 for the solidity.

NOTE. This rule is useful in guaging. A cask in the form of a middle zone of a prolate spheroid, is by guagers called a cask of the first variety.

EXAMPLES.

1. What is the solidity of the middle zone of a prolate spheroid, the diameter *am* of the end being 36, the middle diameter N*n* 60, and the length B*b* 80?

$(2\mathrm{N}n^2 + am^2) \times \mathrm{B}b \times .2618 = 177940.224$, the solid content.

2. Required the solidity of the middle zone of an oblate spheroid, the middle diameter being 100, the diameter of the end 80, and the length 36?

Ans. 248814.72.

3. Required the content in ale gallons of spheroidal cask, whose length is 40 inches; the bung diameter being 32, and head diameter 24 inches. A gallon of ale being 282 cubic inches? *Ans.* 97.44.

4. Required the content in wine gallons of a spheroidal cask, whose length is 20 inches; the bung diameter being 16, and head diameter 12 inches. A gallon of wine being 231 cubic inches?

Ans. 14.869.

§ XIII. *Of a Parabolic* CONOID.

A parabolic conoid, or paraboloid, is a solid, generated by supposing a semiparabola turned about its axis.

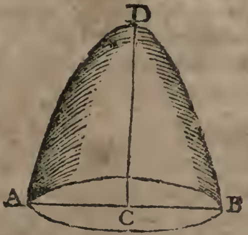
To find the Solidity.

RULE.

Multiply the square of the diameter of its base by the height, and that product by .3927; the last product shall be the solid content.

EXAMPLES.

1. Let ABCD be a parabolic conoid, the diameter of its base, AB, 36 inches, and its height, CD, 33 inches; the solidity is required?



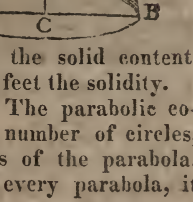
$AB^2 \times CD \times .3927 = 16794.9936$, the solid content which divided by 1728 gives 9.719 feet the solidity.

Demonstration of the rule. The parabolic conoid is constituted of an infinite number of circles, whose diameters are the ordinates of the parabola. Now according to the property of every parabola, it will be,

$$SA : AB :: \overline{AB}^{2} : \frac{\overline{AB}^{2}}{SA} = L, \text{ the } \textit{Latus Rectum}.$$

$$\text{Then} \begin{cases} Sa \times L = \overline{ba}^2 \\ Se \times L = \overline{fe}^2 \\ Sy \times L = \overline{gy}^2, \ \&c. \end{cases}$$

Here $Sa \times L$, $Se \times L$, $Sy \times L$, &c. are a series of terms in arithmetical progression. Therefore $\overline{ba}^2$, $\overline{fe}^2$, $\overline{gy}^2$, &c. are also a series of terms in the same progression, beginning at the point S, wherein $\overline{AB}^2$ is the greatest



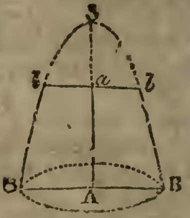
term, and SA, the number of all the terms. Therefore, $\overline{AB}^2 \times \frac{1}{2} SA =$ the sum of all the terms; and circles are to each other as the squares of their radii.

Therefore, if we put $D = AB$, and $H = SA$.

Then $.7854\ DD \times \frac{1}{2} H = .3927\ DDH$ will be the solid content of the conoid; which is just half the cylinder, whose base is $= D$, and height $= H$.

This being rightly understood, it will be easy to raise a theorem for finding the lower frustum of any parabolic conoid.

For supposing $h = aA$, the height of the frustum, and $o = Sa$, the height of the part bSb cut off, then $h+p = SA$, the height of the whole conoid; let $D = BB$ and $d = bb$, then the solidity of BSB will be $D^2h + D^2p; \times .3927$, and the solidity of bSb will be $d^2p \times .3927$, hence the solidity of $BbbB$ must be $D^2h + D^2p - d^2p; \times .3927$, but, by the property of the parabola $SA : Sa :: \overline{AB}^2 : ab^2$, *viz.* $h+p : p :: \frac{D^2}{4} : \frac{d^2}{4}$ hence $p = \frac{hd^2}{D^2 - d^2}$ for p substitute its value in the expression for the *solidity* of the frustum $BbbB$, and it becomes $(D^2h; + D^2 - d^2 \times \frac{hd^2}{D^2 - d^2})$ $\times .3927 = D^2 + d^2 + h \times .3927$.



2. What is the solidity of a parabolic conoid whose height is 84, and the diameter of its base 48?

Ans. 76001.5872.

3. Required the solidity of a paraboloid whose height is 30, and the diameter of its base 40?

Ans. 18849.6.

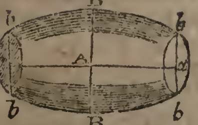
4. Required the solidity of a paraboloid whose height is 22.85, and the diameter of its base 32?

Ans. 9188.55168.

To find the Solidity of the Frustum of a Paraboloid, or Parabolic Conoid.

RULE.

Multiply the sum of the squares of the diameters of the two ends by the height, and that product by .3927 for the solidity.



NOTE. This rule is useful in cask-gauging. For if two equal frustums of a parabolic conoid BbbB, be joined together at their greater bases, they form a cask of the *third* variety. The less diameter of the frustum *bb* will be the head diameter of the cask, and the greater diameter BAB will represent the bung diameter.

EXAMPLES.

1. The greater diameter BAB, of the frustum BbbB of a paraboloid is 32, the less diameter *bb* 26, and the height A *a*, 8; required the solidity?

$(BB^2+bb^2)\times Aa\times .3927=5340.72$, the solidity.

2. Required the solidity of the frustum of a parabolic conoid, whose greater diameter is 30, less diameter 24, and the height 9? *Ans.* 5216.6268.

3. Required the solidity of the frustum of a paraboloid, the diameter of the greater end being 60, that of the less end 48, and the height 18?

Ans. 41733.0144.

4. There is a cask in the form of two equal frustums of a parabolic conoid; the length is 40 inches, the bung diameter 32, and head diameter 24. Required its content in ale gallons? 282 cubic inches being one gallon. *Ans.* 89.1234 gallons.

5. There is a cask in the form of two equal frustums of a paraboloid; the length is 20 inches, the bung

diameter 16, and head diameter 12. Required the content in wine gallons? 231 cubic inches being one gallon. *Ans.* 13.6 gallons.

§ XIV. *Of a Parabolic Spindle.*

A parabolic spindle is formed by the revolution of a parabola about its base, or greatest double ordinate.

To find the Solidity.

RULE.

Multiply the square of the middle diameter by .41888 (being $\frac{8}{15}$ of .7854) and that product by its length; the last product is the solid content.

EXAMPLES.

1. Let ABCD be a parabolic spindle, whose middle diameter CD is 36 inches, and its length AB 99 inches; the solidity is required?

$CD^2 \times AB \times .4188 = 53743.97952$, inches, or 31.10184 feet, the solidity.

2. The length of a parabolic spindle is 60, and the middle diameter 34; what is the solidity?

Ans. 29053.5168.

3. The length of a parabolic spindle is 9 feet, and the middle diameter 3 feet; what is the solidity?

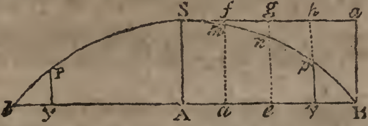
Ans. 33.92928.

4. What is the solidity of a parabolic spindle, whose length is 40, and middle diameter 16?

Ans. 4289.3312.

Demonstration of the rule. A parabolic spindle is constituted of an infinite number of circles, whose diameters are all parallel to the axis SA of the parabola, as *m a*, *n e*, *p y*, *&c.*

Let us suppose the line S*a* parallel to AB, &c.—Then it has already been proved, (section XIV.) that the lines *f m*, *g n*, *h p*, *&c.* are a series of squares, whose roots are in arithmetical progression, consequently their squares, *viz.* fm^2, gn^2, hp^2, *&c.* will be the series of biquadrates, whose roots will be in arithmetical progression: which being premised, we may proceed thus:



1. $SA - fm = ma$
2. $SA - gn = ne$
3. $SA - hp = py$, *&c.* and squaring each equation, we get

1. $\overline{SA}^2 - 2\overline{SA} \times \overline{fm} + \overline{fm}^2 = \overline{ma}^2$

2. $\overline{SA}^2 - 2\overline{SA} \times \overline{gn} + \overline{gn}^2 = \overline{ne}^2$

3. $\overline{SA}^2 - 2\overline{SA} \times \overline{hp} + \overline{hp}^2 = \overline{py}^2$ *&c.*

The sum of these equations will evidently give the sum of the squares of the radii, *m a*, *n e*, *n y*, *&c.* of circles which constitute the solidity of the semi-parabolic spindle. The first terms of the equations being a

series of equal squares, where the number of terms is AB, the sum of the terms will be $\overline{SA^2}\times AB$.

The sum of the second terms of the above equations (*Emerson*'s arithmetic of infinites, Prop. III.) is

$$-2SA\times\frac{SA+AB}{3}=-\frac{\overline{2SA^2}\times AB}{3}$$ wherein SA is the greatest term, and AB the number of terms.

The sum of the third terms of the above equations (*Emerson's* arithmetic of infinites, Prop. V.) is

$$\frac{1}{4+1}\times\overline{SA^2}\times AB=\frac{SA^2\times AB}{5},$$ for $\overline{fm^2}$, $\overline{gn^2}$, $\overline{hp^2}$, &c. is a series of biquadrates, whose greatest term is $\overline{SA^2}$, and number of terms AB. Hence, the sum of all the terms of the above equations will be

$$\overline{SA^2}\times AB;-\frac{\overline{2SA^2}\times AB}{3}+\frac{\overline{SA^2}\times AB}{5}=\frac{\overline{8SA^2}}{15}\times\overline{AB},$$

the sum of all the series of squares, ma^2, ne^2, py^2, &c. But as circles are to each other as the squares of their radii or diameters; it evidently follows that $\frac{8}{15}\overline{SA^2}\times.7854\times AB$ will be the solidity of one-fourth of the spindle ASB, that is $\frac{8}{15}$ of the cylinder S*a*BA circumscribing one-fourth of the spindle; therefore the whole spindle $\frac{8}{15}$ of its circumscribing cylinder: hence if D =the middle diameter, and L the length of the spindle, its solidity will be $\frac{8}{15}\times D^2\times L\times.7854=D^2\times .41888\times L$. Which is the rule.

To find the Solidity of one-fourth of the middle Frustum of the Spindle, SpyA.

Exactly on the same principles as above $\overline{SA^2}\times Ay;-\frac{2SA\times hp}{3}+Ay;+\frac{\overline{hp^2}}{5}\times Ay=$(sum of all the

series of squares $\overline{SA}^2$, $m\,a^2$, $n\,e^2$, and $p\,y^2$;)$=$

$$\left(\overline{SA}^2 - \frac{2SA\times hp}{3} + \frac{\overline{hp}^2}{5}\right)\times Ay = S.$$ Hence $3\overline{SA}^2 - \overline{2SA\times hp} + \frac{3\overline{hp}^2}{5} = \frac{3S}{Ay}$. But $\overline{SA}^2 - \overline{2SA\times hp} + \overline{hp}^2 = \overline{py}^2$, and subtracting this equation from the former we get $2\overline{SA}^2 + \frac{3\overline{hp}^2}{5} - \overline{hp}^2 = \frac{3S}{Ay} - \overline{py}^2$, hence by reduction, &c.

$\overline{2SA}^2 + \overline{py}^2 - \frac{2}{5}\overline{hp}^2$; $\times \frac{1}{3} Ay = S$. And circles are to each other as the squares of their radii, hence

$1.5708\overline{SA}^2 + .7854\overline{py}^2 - 3.1416\overline{hp}^2 \times \frac{1}{3} Ay = S$, or

$\overline{2SA}^2 + \overline{py}^2 - \frac{2}{5}\overline{hp}^2 \times .2618\, Ay = S$, the solidity of one-fourth of the middle frustum of the spindle. Hence

To find the Solidity of the middle Frustum of a Parabolic Spindle.

To twice the square of the middle diameter, add the square of the diameter of the end; and from the sum subtract four-tenths of the square of the difference between these diameters; the remainder multiplied by the length, and that product by .2618 will give the solidity.

Note. This rule is useful in cask-gauging. A cask in the form of the middle frustum of a parabolic spindle, is called by gaugers a cask of the *second* variety; and is the most common of any of the varieties.

EXAMPLES.

1. Required the solidity of the middle frustum of a parabolic spindle EFGH, the length AB being 20, the greatest diameter CD 16, and the least diameter EF or GH 12?

$(2CD^2+GH^2-.4\times(CD-GH)^2)\times AB\times .2618=(512+144-.4\times 4\times 4)\times 20\times .2618=(656-6.4)\times 5.236=649.6\times 5.236=3401.3056$, the solid content.

2. The bung diameter CD of a cask is 32 inches, head diameter EF, 24 inches, and length AB, 40 inches; required its content in ale gallons? 282 cubic inches being 1 gallon. *Ans.* 96.4909 gallons.

3. The bung diameter CD, of a cask is 36 inches, head diameter EF 20 inches, and length AB 36 inches; required its content in wine gallons? 231 cubic inches being 1 gallon. *Ans.* 117.89568 gallons.

To find the Solidity of the middle Frustum of any Spindle, formed by the revolution of a Conic Section about the Diameter of that Section.

RULE.

To the square of the greatest diameter add the square of the least, and four times the square of a diameter taken exactly in the middle between the two; multiply the sum by the length, and that product by .1309 for the solidity.

Note. See the latter part of the demonstration of the rule, in section X.

EXAMPLES.

1. Required the solidity of the middle frustum EFGH of any spindle; the length AB, being 40, the greatest or middle diameter CD, 32, the least diameter EF or GH, 24, and the diameter IK in the middle between GH and CD, 30.157568? *Ans.* 27425.72624.

2. The bung diameter of a cask being 36 inches; head 20; length 36, and a diameter exactly in the middle, 31.95 inches: what is the content in wine gallons? *Ans.* 117 gallons $3\frac{1}{2}$ quarts.

§ XV. *Of the five* Regular Bodies.

A regular, or platonic body, is a solid contained under a certain number of similar and equal plane figures. Only three sorts of regular plane figures joined together can make a solid angle; for three plane angles, at least, are required to make a solid angle, and all the plane angles which constitute the solid angle, must be less, when added together, than four right angles, (*Euclid*, XI. and 21.) Now each angle of an equilateral triangle is 60 degrees; each angle of a square 90 degrees; and each angle of a pentagon 180 degrees. Therefore there can be only five regular bodies, for the solid angles of each must consist either of *three*, *four*, or *five* triangles, *three* squares, or *three* pentagons.

1. The *tetraedron*, or equilateral pyramid, which has four triangular faces; hence all the plane angles about one of its solid angles make 180 degrees.

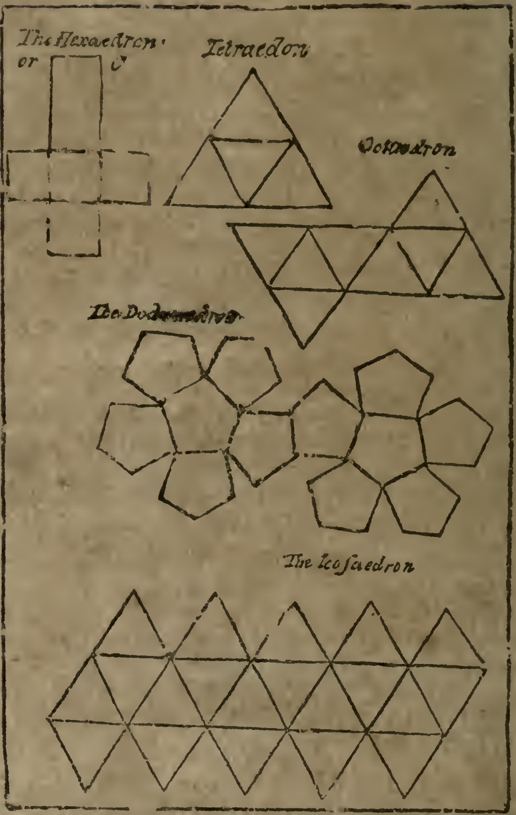
2. The *octaedron*, which has eight equilateral triangular faces; hence all the plane angles about any one of its solid angles make 240 degrees.

3. The *dodecaedron*, which has twelve equilateral pentagonal faces; hence all the angles about any one of its solid angles, make 324 degrees.

4. The *icosaedron*, which has twenty equilateral triangular faces; hence all the angles about any one of its solid angles, make 300 degrees.

5. The *hexaedron*, or cube, which has six equal square faces; hence all the angles about any one of its solid angles make 270 degrees.

Note. If the following figures be exactly drawn on pasteboard, and the lines cut half through, so that the parts may be turned up and glued together, they will represent the five regular bodies above described.


The Hexaedron or Cube
Tetraedon
Octoedron
The Dodecaedron
The Icosaedron

A TABLE, shewing the solidity and superficies of the five regular bodies, the length of a side in each being 1, or unity.

Names of the bodies.	*Solidity.*	*Superficies.*
Tetraedron	.11785113	1.73205081
Octaedron	.47140452	3.46410162
Hexaedron	1.00000000	6.00000000
Icosaedron	2.18169499	8.66025404
Dodecaedron	7.6631189	20.6457288

To find the Solidity or Superficies, of any regular Body, by the Table.

RULE.

1. Multiply the cube of the length of a side of the body, by the tabular solidity, and the product will give the solidity of the body.

2. Multiply the square of the length of a side of the body, by the tabular superficies, and the product will give the superficies of the body.

EXAMPLES.

1. Let ABCD be a *tetraedron*, whose side AB is 12 inches; required the solidity and superficies?

12=AB	Tab. sol. .11785113
12	1728
144=square of AB	94280904
12	23570226
	82495791
1728=cube of AB	11785113
	203.6467526 4 solidity.

And, 144 multiplied by 1.73205081, the tabular superficies, gives 249.41531664 inches, the superficies of the tetraedron.

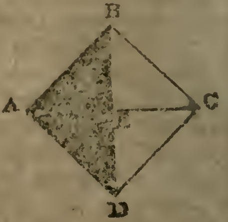
2. Each side of a tetraedron is 3, required its surface and solidity?

Ans. Superficies 15.58845729; solid. 3.18198051

3. Let ABCDE be an *octaedron*, each side being 12 inches; required the solidity and superficies?

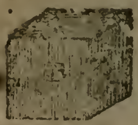
Answer.

814.58701056 inch. solid.
498.83063328 inch. super.

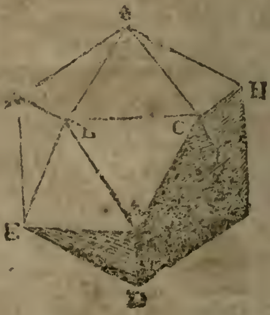


4. Let B be a *hexaedron*, or cube, whose side is 12 inches; required the solidity and superficies?

Ans. $\begin{cases} \text{1728 inches, solidity.} \\ \text{864 inches, superficies.} \end{cases}$



5. Let ABCDEFGHI be an *icosaedron*, each side thereof being 12 inches; required the solidity and superficies?



Ans. $\begin{cases} \text{3769.97470272 inches, solidity.} \\ \text{1247.07658176 inches, superficies.} \end{cases}$

6. Let ABCDEFGHIK be a *dodecaedron*, each side thereof being 12 inches; required the solidity and superficies?

Ans. { 13241.8694592 inches, solidity. 2972.9849472 inches, superficies.

§ XVI. *To measure any* IRREGULAR SOLID.

If you have any piece of wood or stone that is craggy or uneven, and you desire to find the solidity, put the solid into any regular vessel, as a tub, a cistern, or the like, and pour in as much water as will just cover it; then take out the solid, and measure how much the fall of tho water is, and so find the solidity of that part of the vessel.

EXAMPLES.

1. Suppose a piece of wood or stone to be measured, and suppose a cylindrical tub 32 inches diameter, into which let the stone or wood be put, and covered with water: then, when the solid is taken out, suppose the fall of the water 14 inches; square 32, and multiply the square by .7854, the product will be 804.2496, the

area of the base; which multiplied by 14, the depth or fall of the water, the product is 11259.49, &c. which divided by 1728, the quotient is 6.51 feet; and so much is the solid content required.

2. It is reported that *Hiero*, king of *Sicily*, furnished a workman with a quantity of gold to make a crown. When it came home, he suspected that the workman had used a greater alloy of silver than was necessary in its composition, and applied to *Archimedes*, a celebrated mathematician of *Syracuse*, to discover the fraud, without defacing the crown. He procured a mass of gold and another of silver, exactly of the same weight with the crown; considering that if the crown were of pure gold, it would be of equal bulk and displace an equal quantity of water with the golden ball: and if of silver, it would be of equal bulk, and displace an equal quantity of water with the silver ball; but if a mixture of the two, it would displace an intermediate quantity of water.

Now, for example, suppose that each of the three masses weighed 100 ounces; and that on immersing them severally in water, there were displaced 5 ounces of water by the golden, 9 ounces by the silver, and 6 ounces by the crown; then their comparative bulks are 5, 9, and 6.

From 9, silver.	From 6, crown.
Take 6, crown.	Take 5, gold.
3 remainder.	1 remainder.

The sum of these remainders is 4;

Then 4 : 100 :: 3 : 75 ounces of gold.

and 4 : 100 :: 1 : 25 ounces of silver.

That is, the crown consisted of 75 ounces of gold and 25 ounces of silver.

See Cotes' *Hydros. Lect.* p. 81.

CHAPTER III.

The MEASURING of BOARD and TIMBER.

§ I. *Of BOARD MEASURE.*

To find the Superficial Content of a Board or Plank.

RULE.

MULTIPLY the length by the mean breadth. When the board is broader at one end than the other, add the breadths of the two ends together and take half the sum of the mean breadth.

By the Carpenter's Rule.

Set 12 on B to the breadth in inches on A; then against the length, in feet, on B, you will find the superficies on A, in feet.

By Scale and Compasses.

Extend the compasses from 12 to the length in feet, that extent will reach from the breadth, in inches, to the superficies in feet.

EXAMPLES.

1. If a board be 16 inches broad, and 13 feet long; how many feet are contained in it?

Multiply 16 by 13, and the product is 208; which divided by 12, gives 17 feet, and 4 remains, which is a third part of a foot.

By the Carpenter's Rule.

As 12 *on* B : 16 on A : : 13 *on* B : $17\frac{1}{3}$ *on* A.

By Scale and Compasses.

Extend the compasses from 12 to 13, the length in feet, that extent will reach from 16, the breadth in inches, to $17\frac{1}{3}$ the superficies in feet.

2. What is the value of a plank, whose length is 8 feet 6 inches, and breadth throughout 1 foot 3 inches; at $2\frac{1}{2}$d. per foot? *Ans.* 2 shill. $2\frac{1}{2}$d.

3. Required the superficies of a board, whose mean breadth is 1 foot 2 inches, and length 12 feet 6 inches? *Ans.* 14 feet 7 inches.

4. Having occasion to measure an irregular mahogany plank of 14 feet in length, I found it necessary to measure several breadths at equal distances from each other, *viz.* at every two feet. The breadth of the less end was 6 inches, and that of the greater end 1 foot; the intermediate breadths were 1 foot, 1 foot 6 inches, 2 feet, 2 feet, 1 foot, and 2 feet; how many square feet were contained in the plank?

Ans. $19\frac{1}{4}$ feet; —mean breadth $1\frac{3}{8}$ feet, found by dividing the sum of the breadths by their number.

5. Required the value of 5 oaken planks at 3d. per foot, each of them being $17\frac{1}{2}$ feet long; and their several breadths as follows; *viz.* two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{4}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ inches at the narrower? *Ans.* *l.*1 : 5 : $8\frac{1}{4}$.

Having the breadth of a rectangular Plank given in inches, to find how much in length will make a foot, or any other assigned quantity.

Divide 144, or the area to be cut off, by the breadth in inches, and the quotient will be the length in inches.

NOTE. To answer the purpose of the above rule, some carpenters' rules have a little table upon them, in the following form, called

A Table of Board Measure.

0	0	0	0	5	0	$8\frac{1}{2}$	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

By this table you are to understand, that if the breadth be 1 inch, the length must be 12 feet; if 2 inches, the length is 6 feet; if 5 inches broad, the length is 2 feet 5 inches, &c.

If the breadth be not contained in the table on the rule, shut the rule, and look for the breadth in the line of board measure, which runs along the rule from the table of board measure, and over against it on the opposite side, in the scale of inches, is the length required. Thus, if the breadth be 9 inches, you will find the length against 15 inches; if the breadth be 11 inches, you will find the length a little above 13 inches, &c.

EXAMPLES.

1. If a board be 19 inches broad; how many inches in length will make a foot?

Ans. 7.58 inches.

2. From a mahogany plank 26 inches broad, a yard and a half is to be cut off; what distance from the end must the line be struck?

Ans. 74.7692 inches, or 6.23 feet.

§ II. *The Customary Method of* MEASURING TIMBER.

I. The *customary* method of measuring *round* timber, is to gird the piece round the middle with a string; one fourth part of this girt squared and multiplied by the length gives the solidity. Or, if the piece of timber be *squared*, half the sum of the breadth and depth in the middle is considered as the quarter-girt, and used as above.

II. If the piece of timber be very irregular, gird it in several places equally distant from each other, and divide the sum of their circumferences by their number, for a mean circumference; the square of one quarter of the mean circumference, multiplied by the length, will give the solidity.

NOTE. If the circumference be taken in inches, and the length it feet, divide the last product by 144.

III. *Otherwise by the following Table for measuring* TIMBER.

Quart. girt.	*Area.*	*Quart. girt.*	*Area.*	*Quart. girt.*	*Area.*
Inches.	Feet.	Inches.	Feet.	Inches.	Feet.
6	.250	12	1.000	18	2.250
$6\frac{1}{4}$	.272	$12\frac{1}{4}$	1.042	$18\frac{1}{2}$	2.376
$6\frac{1}{2}$	.294	$12\frac{1}{2}$	1.085	19	2.506
$6\frac{3}{4}$	.317	$12\frac{3}{4}$	1.129	$19\frac{1}{2}$	2.640
7	.340	13	1.174	20	2.777
$7\frac{1}{4}$	.364	$13\frac{1}{4}$	1.219	$20\frac{1}{2}$	2.917
$7\frac{1}{2}$	.390	$13\frac{1}{2}$	1.265	21	3.062
$7\frac{3}{4}$	.417	$13\frac{3}{4}$	1.313	$21\frac{1}{2}$	3.209
8	.444	14	1.361	22	3.362
$8\frac{1}{4}$	.472	$14\frac{1}{4}$	1.410	$22\frac{1}{2}$	3.516
$8\frac{1}{2}$	.501	$14\frac{1}{2}$	1.460	23	3.673
$8\frac{3}{4}$	.531	$14\frac{3}{4}$	1.511	$23\frac{1}{2}$	3.835
9	.562	15	1.562	24	4.000
$9\frac{1}{4}$	.594	$15\frac{1}{4}$	1.615	$24\frac{1}{2}$	4.168
$9\frac{1}{2}$	.626	$15\frac{1}{2}$	1.668	25	4.340
$9\frac{3}{4}$	.659	$15\frac{3}{4}$	1.722	$25\frac{1}{2}$	4.516
10	.694	16	1.777	26	4.694
$10\frac{1}{4}$	.730	$16\frac{1}{4}$	1.833	$26\frac{1}{2}$	4.876
$10\frac{1}{2}$	.766	$16\frac{1}{2}$	1.890	27	5.062
$10\frac{3}{4}$	.803	$16\frac{4}{4}$	1.948	$27\frac{1}{2}$	5.252
11	.840	17	2.006	28	5.444
$11\frac{1}{4}$	.878	$17\frac{1}{4}$	2.066	$28\frac{1}{2}$	5.640
$11\frac{1}{2}$	.918	$17\frac{1}{2}$	2.126	29	5.840
$11\frac{3}{4}$	.959	$17\frac{3}{4}$	2.187	$29\frac{1}{2}$	6.044
				30	6.250

The use of the Table.

Multiply the area corresponding to the quarter-girt in inches, by the length of the piece of timber in feet, and the product will be the solidity.

IV. *By the Carpenter's Rule.*

Measure the circumference in the middle of the piece of timber, and take a quarter of it in inches, call this the girt.

Then set 12 on D to the length in feet on C, and against the girt in inches on D, you will find the content in feet on C.

V. *By Scale and Compasses.*

Measure the circumference in the middle of the piece of timber, and take a quarter of it in inches.

Then extend from 12 to this quarter-girt, that extent will reach from the length in feet, being turned twice over, to the solidity.

Note. The buyer is allowed to take the girt any where between the greater end, and the middle of the tree, if it taper.

All branches, or boughs, whose quarter-girt is not less than six inches, are reckoned as timber; and any part of the trunk less than 2 feet in compass is not considered as timber.

An allowance is generally made to the buyer on account of bark; thus for oak, *one-tenth* or *one-twelfth* part of the circumference is deducted; but the allowance for the bark of ash, beech, elm, &c. is a small matter less.

EXAMPLES.

1. If a piece of square timber be 2 feet 9 inches deep, and 1 foot 7 inches broad; and the length 16

feet 9 inches, (or, which is the same thing,* if the quarter-girt be 26 inches, and length 16 feet 9 inches) how many solid feet are contained therein?

```
26 inches, quarter-girt.        16.75=16 feet 9 inch.
26                                 676
---                              -------
156                               10050
52                               11725
---                             10050
676 square.                     -------
                            144)11323.00(78.63 feet.
By the Table.                    -------
                                  1243
     4.694                       -------
     16.75                         910
   -------                        -----
     23470                         460
    32858                         -----
   28164                           28 remainder.
   4694
   -------
   78.62450 feet.
   -------
```

NOTE. The true content, measured as a parallelopipedon, § II. of chap. II. is 72.93 feet. See example 4, page 137.

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* That is, according to the customary method of measuring, mentioned at the beginning of this section. And when the breadth and depth of a piece of timber are nearly equal, this method is nearly true.—Otherwise, *the breadth in the middle, is generally multiplied by the depth in the middle, and that product by the length.*

When the timber tapers regularly, half the sum of the breadths of the two ends is the breadth in the middle, and half the sum of the depths of the two ends is the depth in the middle.

By the Carpenter's Rule.

As 12 *on* D : $16\frac{3}{4}$ *on* C : : 26 *on* D : $78\frac{1}{2}$ *on* C.

By Scale and Compasses.

Extend from 12 to 26, that extent, being turned twice over, will reach from $16\frac{3}{4}$ feet to 78 feet.

2. The quarter-girt of a piece of squared timber is 15 inches, and the length 18 feet, required the solidity? *Ans.* $28\frac{1}{8}$ feet.

3. If a piece of squared timber be 25 inches square at the greater end, and 9 inches square at the less, (or, which is the same thing, the quarter-girt in the middle be 17 inches) and the length 20 feet; how many feet of timber are contained therein? *Ans.* 40.13 feet.

Note. The *true* content, measured as the frustum of a square pyramid, § VII. chap. II. is 43.101 feet. See example 4, page 162.

4. If a piece of squared timber be 32 inches broad and 20 inches deep at the greater end, and 10 inches

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The quarter-girt rule never agrees with this, except the dimensions of a piece of timber in the middle be a true square. In every other case, when the timber is tapering, this rule gives the content *too little*: and the quarter-girt rule is nearer the truth, until six times the square of the quarter-girt, be exactly equal to the sum of the areas of the two ends, together with an area found by multiplying the sum of the breadths of the ends by the sum of their depths; and then it is exactly true. Thus, in the 8th example following, the quarter-girt rule is nearer the truth than the rule given in this note: but when a piece of timber tapers very little, and its breadth differs materially from the depth, the quarter-girt rule ought to be rejected, and this rule in the note should be used instead thereof.

broad and 6 inches deep at the less end, (or, which is the same thing, the quarter-girt in the middle be 17 inches) and the length 18 feet; how many feet of timber are contained therein? *Ans.* 36.11, &c. feet.

NOTE. The *true* content by § VII. chap. II. rule 1, is 37.33 feet. See example 5, page 162.

5. If a piece of round timber be 96 inches in circumference, or the quarter-girt be 24 inches, and the length 18 feet; how many feet of timber are contained therein? *Ans.* 72 feet.

NOTE. The *true* content, measured as a cylinder by § V. chap. II. is 91.67 feet. See example 2, page 149.

6. If a piece of round timber be 86 inches in circumference, or the quarter-girt be $21\frac{1}{2}$ inches, and the length 20 feet; how many feet of timber are contained therein? *Ans.* 64.2 feet.

NOTE. The *true* content, measured as a cylinder is 81.74 feet. See example 3, page 149.

7. If a piece of round timber be 28.2744 inches in circumference at the less end, and 113.0976 inches in circumference at the greater end, (or, which is the same thing, the quarter-girt in the middle be 17.6715 inches) and the length 24 feet; how many feet of timber are contained therein? *Ans.* 52.047 feet.

NOTE. The *true* content, measured as the frustum of a cone, by § VIII. chap. II. is 74.22 feet. *Vide* p. 163.

8. If a piece of timber be 136 inches in circumference at the greater end, and 32 inches in circumference at the less end, (or, which is the same thing, the quarter-girt in the middle be 21 inches) and the length 21 feet; how many feet of timber are contained therein? *Ans.* 64.31 feet.

NOTE. The *true* content, measured as the frustum of a cone, by § VIII. chap. II. 92.34 feet. *Vide* page 163.

9. How many solid feet are contained in a tree whose length is $17\frac{1}{4}$ feet, and girts in five places, *viz.* in the first place 9.43 feet, in the second 7.92, in

in the first place 9.43 feet, in the second 7.92, in the third, 6.15, in the fourth 4.74, and in the fifth 3.16 feet? *Ans.* 42.52 feet.

To find how much in length will make a foot of any squared Timber, of equal thickness from end to end.

Divide 1728, the solid inches in a foot, by the area of the end in inches, and the quotient will be the length of a solid foot, in inches.

Note. To answer the purpose of the above rule, the carpenters' rules sometimes have a little table upon them in the following form, called

A Table of Timber Measure.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the sq.

By this table you are to understand, that if the side of the square be one inch, the length must be 144 feet; if 2 inches be the side of the square, the length must be 36 feet, to make a solid foot.

If the side of the square be not in the little table, look for it in the line of timber measure, running along the rule from the table, and against it, in the line of inches is the length required. Thus, if the side of the square be 16 inches, you will find the length to be 6 inches and 7 tenths, &c.

EXAMPLES.

1. If a piece of timber be 18 inches square, how much in length will make a solid foot?

Ans. $5\frac{1}{3}$ inches.

2. If a piece of timber be 22 inches deep, and 15 inches broad; how much in length will make a solid foot? *Ans.* 5.23 inches.

SCHOLIUM.

The foregoing section contains the whole substance of timber measuring, as practised in all timber yards; and though the method has been noticed as erroneous by various different writers for near a century past, it still stands its ground; nor does it appear probable that it will soon be abolished.

In measuring *round* timber by the foregoing method, of taking a quarter of the circumference in the middle, for the side of a mean square; it is objected that it makes the content *too little*, and that such timber ought to be measured as a *cylinder*, or the *false* content should be increased in the ratio of .0625 to .07958, or as 11 to 14.

But this objection is answered by saying, that before the wood can be squared, and made fit for use, a great part of it goes to waste in chips, and therefore the quantity of round timber ought to be reckoned no more than what the inscribed square will amount to. Now, if the circumference be 1, the area of the section will be .07958, the square of the quarter-girt .0625, and the side of the inscribed square .2251, the square of which is .05067. Therefore to make the content by the inscribed square correspond with the cylindrical content, it ought to be increased in the ratio of .05067 to .07958, or 7 to 11 which is a greater, increase than 11 to 14, and therefore instead of the content by the quarter-girt being *too little*, it is on this consideration *too much*.

Again, when you take the mean girt of a tree, it is very probable that the tree is not perfectly circular in that place, and the more it differs from a circle the greater the quarter-girt will be; so that if you

were to measure such a tree as a *cylinder*, by the proper rule for that purpose, the content would in reality be too much on account of the error in taking the girt.

It is likewise objected that in *round* tapering timber, taking the side of a square in the middle of the piece makes the content *too little*, and that even in a greater proportion than above. The answer given to this objection is, that in almost in all cases the greater must be cut away till it be of the same dimensions as the less end, otherwise the timber cannot be sawn into useful materials ; and therefore there is no just reason for any objection to the rule on this ground; and particularly, as no general rule has hitherto been published, of sufficient merit, to supersede the use of the quarter-girt rule.

Dr. *Hutton*, in the quarto edition of his *Mensuration* (published in 1770) p. 607, has given an easy rule for measuring round timber : thus,

" *Multiply the square of* one-fifth *of the girt or circumference by twice the length, and the product will be the content* (*extremely near the truth.*)"

Dr. *Hutton* says that many reasons may be alledged for changing the customary method of measuring timber, and introducing this rule instead thereof, and the principal reason is, (see p. 614 of the quarto edition of his *Mensuration*) " the preventing of the sellers from playing any tricks with their timber by cutting trees into different lengths, so as to make them measure to more than the whole did ; for, by the false method, this may be done in many respects." Mr. *Bonnycastle* gives the same problems, " to shew the artifices that may be used in measuring timber according to the false method now practised, and the absolute necessity there is for abolishing it."

It is rather singular that neither of those gentlemen should perceive, that the very same tricks and artifices may be practised, with equal success, if Dr.

Hutton's rule be used. The truth of these remarks will easily appear to those who are qualified to read the Doctor's demonstrations; and those who are not, may consult the following

EXAMPLES.

1. Dr. *Hutton*, p. 615, Prob. IV.

Supposing a tree to girt 14 feet at the greater end, 2 feet at the less, and 8 feet in the middle; and that the length be 32 feet.

Rule. If this tree be cut through, exactly in the middle, the two parts will measure to the most possible, by the *common method*, and to more than the whole.

By the common Method.

The whole tree measures	128 feet.
The greater end measures	121 feet.
The less end measures	25 feet.
	Sum 146 exceed-

ing the whole by 18 feet.

By Dr. Hutton's Rule.

The whole tree measures	163.84 feet.
The greater end measures	154.88 feet.
The less end measures	32. feet.
Sum	186.88 exceeding

the whole by 23.04 feet.

Measured as the Frustums of Cones.

The whole tree measures	193.53856 feet.
The great end measures	157.88672 feet.
The less end measures	35.65184 feet.
Sum	193.53856 equal to

the whole, as it ought. U

2. Dr. *Hutton*, p. 616, Prob. V.

Supposing a tree to girt 14 feet at the greater end, 2 feet at the less, and its length to be 32 feet; where must it be cut that the part next the greater end may measure to more than the whole, by the customary measure?

RULE.

Cut it through (if possible) where the girt is $\frac{1}{3}$ of the greatest. This, according to Dr. *Hutton's* rules, will be $7\frac{1}{9}$ feet from the less end, and $24\frac{8}{9}$ feet from the greater end; and the girt at the section will be $\frac{14}{3}$ or $4\frac{2}{3}$ feet.

By the common Method.

The greater end measures	-	$135\frac{41}{81}$ feet.
The whole tree measures	-	- 128 feet.
Diff.		$7\frac{41}{81}$ so that the

part exceeds the whole by 7 feet.

By Dr. Hutton's Rule.

The greater end measures	-	173.44 feet.
The whole tree measures	-	163.84 feet.
Diff.		9.60 so that the

part exceeds the whole by 9.6 feet.

3. Dr. *Hutton*, p. 617, Prob. VI.

Supposing a tree to girt 14 feet at the greater end, 2 feet at the less, and its length to be 32 feet; where must it be cut that the part next the greater end may measure exactly to the same as the whole, by the customary method?

By Dr. *Hutton*'s rules the lengths of the two parts must be 13.599118 feet, and 18.400882 feet; also the girt at the section must be 7.099669.

By the common Method.

The greater end measures - -	128 feet.
The whole tree measures - -	128 feet.

Notwithstanding above $\frac{1}{3}$ part is cut off the length.

By Dr. Hutton's Rule.

The greater end measures -	163.8399 feet,
The whole tree measures - -	163.84 feet,

notwithstanding above $\frac{1}{3}$ part is cut off the length.

The following rules for measuring timber are very accurate, provided the dimensions can be truly taken, and that the timber be in the form of a parallelopipedon, cylinder, frustum of a rectangular pyramid, or the frustum of a cone ; but for the reasons already given, it is not probable that they will ever be brought into general use.

PROBLEM I.

To find the Solidity of Timber Scantling, or Squared Timber, being of equal breadth aud thickness throughout.

RULE.

Multiply the breadth by the thickness, and that product by the length.

Thus, you will find the answer to the first example, p. 211, to be 72.93 feet, or 72 feet 11 inches.

PROBLEM II.

To find the Solidity of a piece of Timber, supposing it to be PERFECTLY *cylindrical.*

RULE.

Multiply the square of one-fourth of the circumference by the length : then say, as 11 is to 14, so is this content, to the true content.—Or, use Dr. *Hutton's* rule, see page 261.

Thus, you will find the answer to the 5th example, page 213, to be 91.63 feet; and to the 6th example 81.62 feet.

PROBLEM III.

To find the Solidity of Squared Timber, tapering regularly.

RULE.

Multiply the breadth at each end by the depth, and also the sum of the breadths by the sum of the depths. These three products added together, and the sum multiplied by one-sixth of the length, will give the solidity.

Thus, you will find the answer to the 3d example, page 212, to be 43.101 feet, and to the 4th example 37.33.

To find the Solidity of round tapering Timber, having the Girt or Circumference of the two ends given in inches, and the length in feet.

RULE.

To the squares of the two circumferences add the square of their sum; multiply this sum by the length, cut off four figures from the right hand for decimals, or move the decimal point four places to the left hand, and $\frac{12}{13}$ of the product will be the content.

Thus, you will find the answer to the 7th example, page 213, to be 74.38 feet, and the 8th example 92.54 feet.

NOTE. By taking the first example in the *scholium*, page 217, you will find

The content of the whole tree	193.931 feet.
The content of the greater end	158.231 feet.
The content of the less end	35.729 feet.
Sum	193.960 equal to

the whole very nearly.

CHAPTER IV.

Of Measuring the Works of the several ARTIFICERS *relating to* BUILDING *; and what Methods and Customs are observed in doing it.*

§ I. *Of* CARPENTERS' *and* JOINERS' *Work.*

THE Carpenters' and Joiners' works, which are measurable, are flooring, partitioning, roofing, wainscoting, &c.

I. *Of Flooring.*

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names according to the position in which they are laid to form a floor; such as trimming joists, common joists, girders, binding joists, bridging joists, and cieling joists.

In boarded flooring, the dimensions must be taken to the extreme parts, and the number of squares of 100 feet must be calculated from these dimensions. Deductions must be made for stair-cases, chimneys, &c.

EXAMPLES.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad; how many squares of flooring are there in that room?

the 17th of July 1819.

By Decimals.	*By Duodecimals.*
57.25	F. I.
28.5	57 : 6
	28 : 3
28625	
45800	456
11450	114
	28 : 7 : 6
100 | 1631.625 feet.	7 : 0 : 0
Squares 16.31625	16 | 31 : 7 : 6

Facit 16 squares and 31 feet.

2. Let a floor be 53 feet 6 inches long, and 47 feet 9 inches broad; how many squares are contained in that floor? *Ans.* 25 squares 54 feet.

3. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad; how many squares are contained in it? *Ans.* 5 squares 98 feet.

4. In a naked floor the girder is 1 foot 2 inches deep, 1 foot broad, and 20 feet long; there are 8 bridging joists, whose *scantlings* (viz. breadths and depths) are 3 inches by $6\frac{1}{2}$ inches, and 20 feet long; 8 binding joists, whose lengths are 9 feet, and scantlings $8\frac{1}{2}$ inches by 4 inches: the cieling joists are 24 in number, each 6 feet long, and their scantlings 4 inches by $2\frac{1}{2}$ inches. Required the solidity of the whole? *Ans.* 72 feet.

5. Suppose a house of three stories, besides the ground floor, was to be floored at *l.*6 10s. per square; the house measures 20 feet 8 inches, by 16 feet 9 inches; there are 7 fire-places, the measures whereof are, two, each of 6 feet by 4 feet 6 inches; two other, each of 6 feet by 5 feet 4 inches; and two, each of 5 feet 8 inches by 4 feet 8 inches; and the seventh, 5 feet 2 inches by 4 feet. The well-hole for the stairs is 10 feet 6 inches by 8 feet 9 inches. What will the whole come to? *Ans.* *l.*53. 13s. $3\frac{1}{4}$d.

II. *Of Partitioning.*

Boarded partitions are measured by the square in the same manner as flooring, and deductions must be made for doors and windows, except they are included by agreement.

The strongest partitions are those made with framed timber, all the parts of which are measured as in flooring, except the king-posts, and these are of the same kind as in roofing, an example of which will be given in the next article.

EXAMPLES.

1. If a partition between rooms be in length 82 feet 6 inches, and in height 12 feet 3 inches; how many squares are contained therein?

Ans. 10 squares 10 feet.

2. If a partition between rooms be in length 91 feet 9 inches, and its breadth 11 feet 3 inches; how many squares are contained in it?

Ans. 10 squares 32 feet.

III. *Of Roofing.*

In roofing take the whole length of the timber for the length of the framing, and for the breadth gird over the ridge from wall to wall with a string. This length and breadth must be multiplied together for the content.

It is a rule also among workmen, that the flat of any house, and half the flat thereof, taken within the walls, is equal to the measure of the roof of the same house; but this is when the roof is of a true pitch. The pitch of every roof ought to be made according to its covering, which in *England* is lead, pantiles, plaintiles, or slates. The usual pitches, are the pedi-

ment pitch, used when the covering is lead; the perpendicular height is $\frac{2}{9}$ of the breadth of the building. The common, or true pitch, where the length of the rafters are $\frac{3}{4}$ of the breadth of the building; this is used when the covering is plaintiles. The Gothic pitch, is when the length of the principal rafters is equal to the breadth of the building, forming an equilateral triangle: This pitch is used when the covering is of pantiles.

In the measuring of roofing for workmanship alone, all holes for chimney-shafts and sky-lights are generally deducted. But in measuring for work and materials, they commonly measure in all sky-lights, luthern-lights, and holes for the chimney-shafts, on account of their trouble, and waste of materials.

EXAMPLES.

1. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad; how many squares of roofing will cover that house?

By Decimals.

```
      18.25
       44.5
      -----
       9125
      7300
     7300
    -------
Flat 812.125
Half 406.
    -------
 100 | 12  18
    -------
      12.18
```

By Duodecimals.

```
           F.  I.
           44 : 6
           18 : 3
          --------
          352
          44
          11 : 1 : 6
           9 : 0 : 0
          ----------
The flat  812 : 1 : 6
The half  406
          ------
      Sum 12|18
```

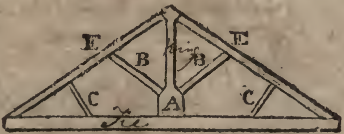
Facit 12 squares 18 feet.

2. What cost the roofing of a house at 10s. 6d. per square; the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches; the roof being of a true pitch? *Ans.* $l.12$ 12s. $11\frac{3}{4}$d.

3. The roof of a house is of a true pitch; and the house measures 40 feet 6 inches in length, within the walls, and 20 feet 6 inches in breadth; how many squares of roofing are contained therein?

Ans. 12.45375.

NOTE. All timbers in a roof are measured in the same manner as in floors, except king-posts, &c. such as A in the annexed figure, where there is a necessity for cutting out parallel pieces of wood from the sides, in order that the ends of the braces B, that come against them, may have what the workmen call a square butment. *To measure king-posts for workmanship,* take their breadth and depth at the widest place, and multiply these together, and the product by the length. *To find the quantity of materials,* if the pieces sawn out are $2\frac{1}{2}$ inches broad, or upwards, and more than 2 feet long, they are considered as pieces of timber fit for use.



In measuring these pieces, the shortest length must always be taken, because the sawing of them from the king-post, renders a part of them useless. The solidity of these must be deducted from the solidity of the king-post. When the pieces sawn out are of smaller dimensions than above described, the whole post is measured as solid without any deduction, the pieces cut out being esteemed of little or no value.

4. Let the tie-beam D, in the above figure, be 36 feet long, 9 inches broad, and 1 foot 2 inches deep; the king-post A 11 feet 6 inches high, 1 foot broad

at the bottom, and 5 inches thick; out of this post are sawn two equal pieces from the sides, each 7 feet long and 3 inches broad. The braces B, B, are 7 feet 6 inches long, and 5 inches by 5 inches square: the rafters E, E, are 19 feet long, 5 inches broad, and 10 inches deep: the struts, C, C, are 3 feet 6 inches long, 4 inches broad, and 5 inches deep. Required the measurement for workmanship, and likewise for materials?

Answer.

F.	I.	P.	
31	6	0	solidity of the tie-beam D.
4	9	6	solidity of the king-post A.
2	7	3	solidity of the braces B, B.
13	2	4	solidity of the rafters E, E.
—	11	8	solidity of the struts C, C.
53	0	9	solidity for workmanship.
1	5	6	solidity cut from the king-post.
51	7	3	solidity for materials.

IV. *Of Wainscoting, &c.*

Wainscoting is measured by the yard square, consisting of 9 square feet. The dimensions are taken in feet and inches; thus, in taking the height of a room they girt over the cornice, swelling pannels, and mouldings, with a string; and for the compass of the room, they measure round the floor. Doors, window-shutters, and such like, where both sides are planed, are considered as work and half; therefore in measuring a room they need not be deducted, but the room may be measured as if there were none; then the contents of the doors and shutters must be found, and the half thereof added to the content of the whole room.

Windows, where there are no shutters, must be deducted; also chimneys, window-seats, cheek-boards, sopheta-boards, linings, &c. must be measured by themselves.

Weather-boarding is measured by the yard square, and sometimes by the square.

Windows are generally made and valued by the foot superficial measure, and sometimes at so much per window. When they are measured, the dimensions must be taken in feet and inches, from the under side of the sill to the upper side of the top-rail, for the height; and for the breadth, from outside to outside of the jambs. This length multiplied by the breadth will be the superficies.

Stair-cases are measured by the foot superficial, and the dimensions are taken with a string, girt over the *riser* and *tread;* and that length, or girt, multiplied by the length of the step, will give the superficies.

The rail is taken at so much per foot in length, according to the diameter of the well-hole; architrave string-boards by the foot superficial; brackets and strings at so much per piece, according to the workmanship.

Door-cases are measured by the foot superficial, and the dimensions must be taken with a string girt round the architrave and inside of the jambs, for the breadth; and for the length, add the length of the two jambs to the length of the cap-piece (taking the breadth of the opening for the length) the product of these two will be the superficies.

Frame doors are measured by the foot, or sometimes by the yard square.

Modillion cornices, coves, &c. are generally measured by the foot superficial.

Frontispieces are measured and valued by the foot superficial, and every part is measured separately, *viz*, architrave, frieze, and cornice.

EXAMPLES.

1. If a room, or wainscot, being girt downwards over the mouldings, be 15 feet 9 inches high, and 126 feet 3 inches in compass; how many yards does that room contain?

By Duodecimals.	*By Decimals.*
F. I.	
126 : 3	126.25
15 : 9	15.75
630	63125
126	88375
63 : 1 : 6	63125
31 : 6 : 9	12625
3 : 9 : 0	9)1988.4375
9)1988 : 5 : 3	220.8
Ans. 220.8	*Ans.* 220 yards 8 feet.

2. If a room of wainscot be 16 feet 3 inches high, and the compass of the room 137 feet 6 inches; how many yards are contained in it?

Ans. 248 yards 2 feet.

3. If the window-shutters about a room be 69 feet 9 inches broad, and 6 feet 3 inches high, how many yards are contained therein at work and half?

Ans. 72.656 yards.

4. What will the wainscoting of a room come to at 6 shillings per square yard, supposing the height of the room, including the cornice and moulding, be 12 feet 6 inches, and the compass 83 feet 8 inches; three window-shutters, each 7 feet 8 inches by 2 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the shutters and door being worked on both sides, are reckoned work and half? *Ans.* l.36 4s. $6\frac{2}{3}$d.

5. A rectangular room measures 129 feet 5 inches round, and is to be wainscoted at 3s. 6d. per square yard. After due allowance for girt of cornice, &c. it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9 inches; the window shutters, two pair are 7 feet 3 inches by 4 feet 6 inches; the cheek-boards round them come 15 inches below the shutters, and are 14 inches in breadth; the lining-boards round the door-way are 16 inches broad; the door and window-shutters being worked on both sides, are reckoned as work and half, and paid for accordingly: the chimney 3 feet 9 inches by 3 feet, not being enclosed, is to be deducted from the superficial content of the room. The estimate of the charge is required? *Ans.* *l.*43 : 4s. : 6¾d.

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§ II. *Of Bricklayers' Work.*

The principal is tiling, walling, and chimney-work.

I. *Of Tiling.*

Tiling is measured by the square of 100 feet, as flooring, partitioning, and roofing were in the carpenter's work; so that between the roofing and tiling, the difference will not be much; yet the tiling will be the most; for the bricklayers sometimes will require to have double measure for hips and vallies. When gutters are allowed double measure, the way is to measure the length along the ridge-tile, and by that means the measure of the gutters becomes double: it is usual also to allow double measure at the eaves, so much as the projection is over the plate, which is commonly about 18 or 20 inches.

EXAMPLES.

1. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 37 feet 3 inches, and the length 45 feet; I demand how many squares of tiling are contained therein?

By Duodecimals.	*By Decimals.*
F. I.	37.25
37 : 3	45
45 : 0	——
——	18625
185	14900
148	——
11 : 3	16 | 76.25
——	
16|76 : 3	

Ans. 16 squares, 76 feet.

2. There is a roof covered with tiles, whose depth on both sides (with the allowance at the eaves) is 35 feet 9 inches, and the length 43 feet 6 inches; I demand how many squares of tiling are in the roof?

Ans. 15 squares 55 feet.

3. What will the tiling a barn cost at *l.*1 5s. 6d. per square, the length being 43 feet 10 inches, and breadth 27 feet 5 inches, on the flat, the eaves projecting 16 inches on each side? *Ans. l.*24 9s. $5\frac{1}{2}$d.

II. *Of Walling.*

Bricklayers commonly measure their work by the rod square of 16 feet and an half; so that one rod in length, and one in breadth, contain 272.25 square feet; for 16.5, multiplied into itself, produces 272.25 square feet. But in some places the custom is to allow 18

feet to the rod; that is, 324 square feet. And in some places the usual way is, to measuré by the rod of 21 feet long and 3 feet high, that is, 63 square feet; and here they never regard the thickness of the wall in measuring, but regulate the price according to the thickness.

When you measure a piece of brick-work, the first thing is to enquire by which of these ways it must be measured, then, having multiplied the length and breadth in feet together, divide the product by the proper divisor, *viz.* 272.25; 324; or 63; according to the measure of the rod, and the quotient will be the answer in square rods of that measure.

But commonly brick-walls, that are measured by the rod, are to be reduced to a standard thickness; *viz.* of a brick and a half thick (if it be not agreed on the contrary;) and to reduce a wall to standard thickness, this is

THE RULE.

Multiply the number of superficial feet that are found to be contained in any wall by the number of half-bricks which that wall is in thickness; one-third part of that product shall be the content in feet, reduced to the standard thickness of one brick and a half.

EXAMPLES.

1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick; how many rods of brick-work are contained therein, when reduced to the standard?

By Decimals.

```
         19.25
          72.5
       -------
          9625
         3850
       13475
       -------
      1395.625
            11
      --------
    3)15351.875
    -----------
272.25)5117.291(18 rods.
       ------
       239479
       ------
  68.06)216.79(3 quar.
        ------
        12.61 feet.
```

By Duodecimals.

```
    F.  I.
    72 : 6
    19 : 3
   -------
   648
   72
     18 : 1 : 6
      9 : 6 : --
   -------------
   1395 : 7 : 6
             11
   -------------
3)15351 : 10 : 6
-----------------
272)5117 : (18 rods.
   -----
    2397
   -----
68)221(3 quarters.
   ---
    17 feet.
```

Note. That 68.06 is one-fourth part of 272.25, and 68 is one-fourth of 272.

In reducing feet into rods, it is usual to reject the odd parts, and divide only by 272, as is done in the second way of the last example; here the answer is 18 rods 3 quarters and 17 feet; about $4\frac{1}{2}$ feet more than by the first way, where it is done decimally; a difference too trifling to be considered in practice.

To find proper Divisors for bringing the Answer in feet, or rods, of the Standard thickness; without multiplying the Superficies, by the number of half-bricks, &c.

Divide 3, the number of half-bricks in $1\frac{1}{2}$, by the number of half-bricks in the thickness, the quotient will be a divisor, which will give the answer in feet.

But if you would have a divisor to bring the answer in rods at once, multiply 272 by the divisor found for feet, and the product will be a divisor for rods; as in the following table.

The thickness of the wall.	Divisors for the answer in feet.	Divisors for the answer in rods.
1 Brick	1.5	408.
$1\frac{1}{2}$ Brick	1.	272.
2 Bricks	.75	204.
$2\frac{1}{2}$ Bricks	.6	163.2
3 Bricks	.5	136.
$3\frac{1}{2}$ Bricks	.4285	116.6
4 Bricks	.375	102.
$5\frac{1}{2}$ Bricks	.2727	74.17

By Scale and Compasses.

Extend the compasses from the tabular divisor against the given thickness, to the length of the wall, that extent will reach from the breadth to the content.

Or by the Carpenter's Rule.

As the tabular divisor, against the thickness of the wall, is to the length of the wall; so is the breadth to the content.

Taking the preceding example, extend the compasses from 74.17 to 72.5, that extent will reach from 19.25 to $18\frac{3}{4}$ rods.

By the Carpenter's Rule.

$$\begin{array}{ccccccc} B & & A & & B & & A \\ 74.17 & : & 72.5 & :: & 19.25 & : & 18\frac{3}{4} \end{array}$$

The dimensions of a building are generally taken by measuring half round the outside, and half round the inside, for the whole length of the wall; this length being multiplied by the height gives the superficies. And to reduce it to the standard thickness. &c. proceed as above. All the vacuities, such as doors, windows, window-backs, &c. must be deducted.

To measure an arched-way, arched-window, or doors, &c. take the height of the window, or door, from the crown or middle of the arch, to the bottom or sill; and likewise from the bottom or sill to the spring of the arch, that is, where the arch begins to turn. Then to the latter height add twice the former, and multiply the sum by the width of the window, door, &c. and one-third of the product will be the area, sufficiently near for practice.

2. If a wall be 245 feet 9 inches long, 16 feet 6 inches high, and two bricks and a half thick; how many rods of brick-work are contained therein, when reduced to the standard thickness.

Ans. 24 rods 3 quarters 24 feet.

3. How many rods are contained in a wall $63\frac{1}{2}$ feet long, 14 feet 11 inches high, and $2\frac{1}{2}$ bricks in thickness, when reduced to the standard?

Ans. 5 rods 218 feet.

4. A triangle gable-end is raised to the height of 15 feet above the end-wall of a house, whose width is 45 feet, and the thickness of the wall is $2\frac{1}{2}$ bricks; required the content in rods at standard thickness?

Ans. 2 rods 18 feet.

5. Admit the end-wall of a house to be 28 feet 10 inches in breadth, and the height of the roof from the

ground 55 feet 8 inches, the gable (or triangular part above the side walls) to rise 42 courses of bricks, reckoning 4 courses to a foot; and that 20 feet high be $2\frac{1}{2}$ bricks thick, 20 feet more 2 bricks thick, and the remaining 15 feet 8 inches $1\frac{1}{2}$ brick thick; what will the work come to at *l.*5 16s. per rod, the gable being one brick in thickness? *Ans. l.*48 13s. $5\frac{3}{4}$d.

III. *Of Chimneys.*

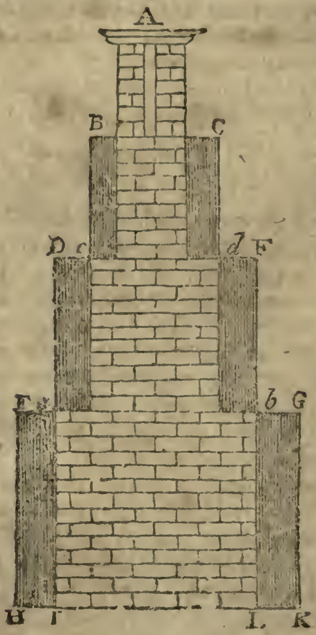
If you are to measure a chimney standing by itself, without any party-wall being adjoining, then girt it about for the length, and the height of the story is the breadth; the thickness must be the same as the jambs are of, provided that the chimney be wrought upright from the mantle-tree to the cieling, not deducting any thing for the vacancy between the floor (or hearth) and the mantle-tree, because of the gatherings of the breast and wings, to make room for the hearth in the next story.

If the chimney-back be a party-wall, and the wall be measured by itself, then you must measure the depth of the two jambs, and the length of the breast for a length, and the height of the story for the breadth, at the same thickness your jambs were of.

When you measure chimney-shafts, *viz.* that part which appears above the roof, girt them with a line round about the least place of them, for the length, and take the height for the breadth: and if they be four inches thick, then you must set down their thickness at one brick-work; but if they be wrought 9 inches thick (as sometimes they are, when they stand high and alone above the roof,) then you must account your thickness $1\frac{1}{2}$ brick, in consideration of plastering, and the trouble in scaffolding.

It is customary, in most places, to allow double measure for chimneys.

EXAMPLES.



1. Suppose the figure, ABCDEFGHIK, to be a chimney that hath a double funnel towards the top, and a double shaft, ABC. Then suppose in the parlour, the breast IL and the two jambs HI and LK to measure 18 feet 9 inches, and the height of the room HF, or I*a*, to be 12 feet 6 inches;—in the first floor let the breast and the two jambs, *viz. ab*, girt 14 feet 6 inches, and the height *a* D, be 9 feet;—in the second

floor let the breast and the jambs, *viz. cd*, girt 10 feet 3 inches, and the height c B be 7 feet;—above the roof, let the compass of the shaft ABC be 13 feet 9 inches, and its height AB 6 feet 6 inches; lastly, let the length of the middle partition, which parts the funnels, be 12 feet, and its thickness 1 foot 3 inches: how many rods of brick-work, standard measure, are contained in the chimney, allowing double measure?

```
        F.  I.
   1.  18 : 9
       12 : 6
      -------
      225 : 0
        9 : 4 : 6
      -----------
FGHK  234 : 4 : 6
      -----------
        F.  I.
   2.  14 : 6
        9 : -
      -------
Da Eb 130 : 6
      -------
        F.  I.
   3.  10 : 3
        7 : -
      -------
Bc Cd  71 : 9
      -------
        F.  I.
   4.  13 : 9
        6 : 6
      -------
       82 : 6
        6 : 10 : 6
      ------------
ABC    89 : 4  : 6
      ------------
```

```
      F.  I.
       1 : 3
      12 : -
     -------
      15 : 0 partition.
     -------
      F.   I. Pts.
     234 : 4 : 6 parlour.
     130 : 6 : - first floor.
      71 : 9 : - sec. floor.
      89 : 4 : 6 shaft.
      15 : - : - partition.
     ----------------
Feet 541 : - : - sum.
     ----------------
Feet 1082 : - : - double.
     ----------------
     272)1082(3 rods.
         ----
        68)266(3 quarters.
           ---
           62   feet.
           --
```

Ans. 3 rods, 3 quarters, and 62 feet, admitting the common thickness to be $1\frac{1}{2}$ brick.

Note. This chimney being measured as if it were solid, no vacuity, or opening, for the fire-place, in

any of the floors, is drawn in the above figure. Also the jambs HIFa, LKGb, are supposed to be perpendicular to the breast IL, *viz.* HIL and ILK are right angles; the same must be observed in the other two floors.

2. Suppose the breadth IL of the breast of a chimney, not standing in an angle, to be 7 feet 3 inches, the depth HI or LK of the jambs 3 bricks thick; the height of the room Id 14 feet 6 inches; the vacuity for the fire-place 4 feet high, 3 feet 6 inches wide, and 3 bricks deep. The chimney and fire-place in each of the other two rooms are of the same dimensions, but the height of the room aD is 12 feet; and the jamb cB, which is the same height as the story, 10 feet 6 inches. The shaft BA stands 6 feet above the roof, and its compass is 10 feet; also its thickness is estimated at $1\frac{1}{2}$ brick; how many rods of brickwork standard thickness, allowing double measure, are contained in the chimney, deducting the three fire-places. *Ans.* 3 rods 3 quarters 5 feet.

§ III. *Of Plasterers' Work.*

The plasterers' works are principally of two kinds; namely, 1. plastering upon laths, called cieling, and 2. plastering upon walls, or partitions made of framed timber, called rendering. In plastering upon walls, no deductions are made except for doors, windows, &c. but in plastering timber partitions, in large warehouses, &c. where several of the braces and larger timbers project from the plastering, a fifth part is commonly deducted. Plastering between these timbers is generally called rendering between quarters.

Whitening and colouring are measured in the same manner as plastering: as in rendering between these projecting timbers one fifth-part is deducted, so in co-

louring, it will be necessary to add one-fourth or one-fifth of the whole, for the trouble of colouring the projections.

Plasterers' work is measured by the yard square, consisting of 9 square feet. In arches, the girt round them, multiplied by the length, will give the superficies.

EXAMPLES.

1. If a cieling be 59 feet 9 inches long, and 24 feet 6 inches broad; how many yards does that cieling contain?

By Duodecimals.

F.	I.	
59	9	
24	6	
236		
118		
29	10	6
18	0	0
9)1463	10	6

162 yards 5 feet.

By Decimals.

59.75
24.5

29875
23900
11950

9)1463.875 feet.

Ans. 162.65 yards.

2. If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high; how many yards are in those partitions? *Ans.* 176.87 yards.

3. What will be the expence of plastering a cieling, at $11\frac{1}{2}$d. per yard, supposing the length 22 feet 7 inches, and breadth 13 feet 11 inches?

*Ans. l.*1 13s. $5\frac{1}{2}$d.

4. There is a partition which measures 234 feet 8 inches round, and 14 feet 6 inches high, this partition is rendered between quarters, that is, it is made of framed timber, and the interstices are filled up with

lath and plastering. The lathing and plastering will be 8d. per yard, and the whitening 2d. per yard; what will the whole come to?

Ans. $l.$13 17s. $2\frac{3}{4}$d.

5. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches, to the under side of the cornice, which projects 5 inches from the wall, on the upper part next the cieling; required the quantity of rendering and plastering, there being no deductions but for one door, the size whereof is 7 feet by 4?

Ans. 53 yards 5 feet of rendering, and 18 yards 5 feet of cieling.

6. The circular vaulted roof of a church measures 105 feet 6 inches in the arch, and 275 feet 5 inches in length; what will the plastering come to at 1s. per yard? *Ans.* $l.$150 17s. $5\frac{3}{4}$d.

161 - 8 - 3 $\frac{}{4}$

§ IV. *Of Painters' Work.*

The taking the dimensions of painters' work is the same as that of joiners, by girting over the mouldings and swelling pannels in taking the height; and it is but reasonable that they should be paid for that on which their time and colour are both expended. The dimensions thus taken, the casting up, and reducing feet into yards, is altogether the same as the joiners' work, but the painter never requires work and half, but reckons his work once, twice, or thrice coloured over. Only take notice, that window-lights, window-bars, casements, and such-like things, they do at so much a piece.

EXAMPLES.

1. If a room be painted, whose height (being girt over the mouldings) is 16 feet 6 inches, and the com-

pass of the room 97 feet 9 inches; how many yards are in that room?

By Duodecimals.	*By Decimals.*
F. I.	
97 : 9	97.5
16 : 6	16.5
584	48875
98	58650
48 : 10 : 6	9775
9)1612 : 10 : 6	9)1612.875
179 : 1	179.2.

Facit 179 yards 2 feet, nearly.

2. A gentleman had a room painted at $8\frac{1}{2}$d. per yard, the measure whereof is as follows; the height 11 feet 7 inches, the compass 74 feet 10 inches, the door 7 feet 6 inches by 3 feet 9 inches; five window-shutters, each 6 feet 8 inches by 3 feet 4 inches, the breaks in the windows 14 inches deep and 8 feet high; the opening for the chimney 6 feet 9 inches by 5 feet, to be deducted; the shutters and doors are coloured on both sides; what will the whole come to?

Ans. *l.*4 6s. 11d.

3. Suppose a room were to be painted, and that its length is 24 feet 6 inches, breadth 16 feet 3 inches, and height 12 feet 9 inches; also the size of the door 7 feet by 3 feet 6 inches, and the size of the window-shutters to each of the windows, there being two, is 7 feet 9 inches by 3 feet 6 inches; but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep; what will be the expence of giving it 3 coats, at 2d. per yard each; the size of the fire-place to be deducted, being 5 feet by 5 feet 6 inches?

Ans. *l.*3 3s. $10\frac{1}{2}$.

§ V. *Of Glaziers' Work.*

Glaziers take their dimensions in feet, inches, and eighths or tenths, or else in feet and hundredth parts of a foot, and estimate their work by the square-foot.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But more generally, they measure the length and breadth of the window over all the panes and their frames, for the length and breadth of the glazing.

Circular or oval windows, as fan lights, &c. are measured as if they were square, taking for their dimensions the greatest length and breadth; as a compensation for the waste of glass, and labour in cutting it to the necessary forms.

Plumbers' work is rated at so much per pound, or by the hundred-weight of 112 pounds.

Sheet-lead, used in roofing, guttering, &c. weighs from 7 to 12 pound per square foot, according to the thickness. And a pipe of an inch bore weighs commonly 13 or 14 pounds per yard in length.

EXAMPLES.

1. If a pane of glass be 4 feet 8 inches and 3 quarters long, and 1 foot 4 inches and 1 quarter broad; how many feet of glass are in that pane?

By Duodecimals.	*By Decimals.*
F. I. P.	4.729
4 : 8 : 9	1.354
1 : 4 : 3	
	18916
4 : 8 : 9	23645
1 : 6 : 11 : 0	14187
1 : 2 : 2 : 3	4729
6 : 4 : 10 : 2 : 3	6.403066

Ans. 6 feet 4 inches.

2. If there be 8 panes of glass each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad; how many feet of glass are contained in the said 8 panes? *Ans.* 53 feet 5 inches.

3. If there be 16 panes of glass, each 4 feet 5 inches and a half long, and 1 foot 4 inches 3 quarters broad; how many feet of glass are contained in them? *Ans.* 99 feet 6 inches.

4. If a window be 7 feet 3 inches high, and 3 feet 5 inches broad; how many square feet of glazing are contained therein? *Ans.* 24 feet 9 inches.

5. There is a house with three tiers of windows, 7 in a tier; the height of the first tier is 6 feet 11 inches, of the second 5 feet 4 inches, and of the third 4 feet 3 inches; the breadth of each window is 3 feet 6 inches; what will the glazing come to at $14\frac{1}{2}$d. per foot? *Ans. l.*24 8s. $5\frac{1}{2}$d.

6. What will the glazing a triangular sky-light come to at 10d. per foot: the base being 12 feet 6 inches long, and the perpendicular height 16 feet 9 inches? *Ans. l.*4. 7s. $2\frac{3}{4}$d.

7. What is the area of an elliptical fan-light, of 14 feet 6 inches in length, and 4 feet 9 inches in breadth? *Ans.* 68 feet 10 inches.

8. What cost the covering and guttering a roof with lead, at 18s. per hundred-weight; the length of the roof being 43 feet, and the girt over it 32 feet; the guttering being 57 feet in length and 2 feet in breadth; admitting a square foot of lead to weigh $8\frac{3}{4}$ pounds. *Ans. l.*104 15s. $3\frac{3}{4}$d.

§ VI. *Of Masons' Work.*

Masons measure their work sometimes by the foot solid, sometimes by the foot superficial, and sometimes by the foot in length. In taking dimensions they girt all their mouldings as joiners do.

The solids consist of blocks of marble, stone, pillars, columns, &c. The superficies are pavements, slabs, chimney-pieces, &c.

Masons reckon all such stones as are above two inches thick, at so much per foot, solid measure. And for the workmanship, they measure the superficies of that part of the stone which appears without the wall.

EXAMPLES.

1. If a wall be 97 feet 5 inches long, 18 feet 3 inches high, and 2 feet 3 inches thick; how many solid feet are contained in that wall?

By Duodecimals.	*By Decimals.*
F. I.	
97 : 5 length.	97.417
18 : 3 breadth or height.	18.25
776	487085
97	194834
24 : 4 : 3	779336
6 : 0 : 0	97417
1 : 6 : 0	
	1777.86025
1777 : 10 : 3 superficies.	2.25
2 : 3 thickness.	
	888930125
3555 : 8 : 6	355572050
444 : 5 : 6 : 9	255572050
4000 : 2 : 0 : 9 solidity.	4000.1855625

2. If a wall be 107 feet 9 inches long, and 20 feet 6 inches high; how many superficial feet are contained therein? *Ans.* 2208 feet 10 inches.

3. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high; how many superficial rods of 63 square feet each, are contained therein?

Ans. 29 rods 25 feet.

4. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 6s. per foot superficial? *Ans.* *l.*3 1s. 5d.

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§ VII. *Of Paviors' Work.*

Paviors' work is measured by the square yard, consisting of 9 square feet. The superficies is found by multiplying the length by the breadth.

EXAMPLES.

1. What cost the paving a foot path at 2s. 4d. per yard; the length being 35 feet 4 inches, and breadth 8 feet 3 inches?

By Decimals.	*By Duodecimals.*
	F. I.
35.3383 &c.	35 : 4
$8\frac{1}{4}$	8 : 3
282.6666	282 : 8
8.8333	8 : 10
9)291.4999 &c.	9)291 : 6
3s. 4d. is $\frac{1}{6}$)32.3888 &c.	3s. 4d. is $\frac{1}{6}$)32.3 : 6
*l.*5.3981	Feet - - - 5 6 8
20	3=$\frac{1}{3}$ of a yard 0 1 $1\frac{1}{4}$33
	6=$\frac{1}{6}$ of 3 feet 0 0 2.88
s. 7.9620	
12	*l.*5 7 $11\frac{1}{2}$.21
d. 11.5440	
4	
2.1760	

*Answer l.*5 7s. $11\frac{1}{2}$d.

2. What will the paving a court-yard come to at 3s. 4d. per yard, the length being 24 feet 5 inches, and breadth 12 feet 7 inches? *Ans. l.*5 13s. $9\frac{1}{2}$d.

3. What will be the expence of paving a rectangular court-yard, its length being 62 feet 7 inches, and breadth 44 feet 5 inches; and in which there is laid a foot-path the whole length of it, $5\frac{1}{2}$ feet broad, with flat stones, at 3s. per yard, the rest being paved with pebbles at 2s. 6d. per yard? *Ans. l.*39 11s. $3\frac{1}{4}$d.

CHAPTER V.

Of Gauging.

GAUGING is the art of measuring and finding the contents of all sorts of vessels, in gallons, or cubic inches; such as casks, brewers' vessels, &c. &c.

Of the Sliding Rule.

The sliding rule is an instrument, particularly useful in gauging, made generally of box, in the form of a parallelopipedon. There are various kinds, but the the most convenient, or at least that which is most used in the excise, was invented by Mr. *Verie*, collector of the excise.

1st, The line, marked A, on the face of this rule, is called *Gunter's Rule*, and is numbered 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. At 2150.42 is a brass pin, marked MB, signifying the cubic inches in a bushel of malt; at 282 is another brass pin, marked A, signifying the number of cubic inches in a gallon of ale.

2dly, The line marked B is on the slide, and is divided in exactly the same manner as that marked A; there is another slide B, which is used along with this; the two brass ends are then placed together, and so make a double radius numbered from the left-hand towards the right. At 231, on the second radius, is a

brass pin, marked W, signifying the cubic inches in a gallon of wine; at 314 is another brass pin, marked C, signifying the circumference of a circle whose diameter is 1. The manner of reading, and using these lines, is exactly the same as the lines A and B, described in chap. XI. page 52, &c.

3dly, The back of the first radius, or slide, marked B, contains the divisors for *ale, wine, mash-tun gallons, malt, green starch, dry starch, hard soap hot, hard soap cold, green soft soap, white soft soap, flint glass, &c. &c.* as in the table, prob. 1, following.—*The back of the second radius, or slide, marked* B, contains the gauge-points correspondent to these divisors, where S stands for squares, and C for circles.

4thly, The line M. D. on the rule, signifying malt depth, is a line of numbers beginning at 2150.42, and is numbered from the left to the right-hand 2. 10. 9. 8. 7. 6. 5. 4. 3. This line is used in malt gauging.

5thly, The two slides, B, just described, are always used together either with the line A; MD; or the line D, which is on the opposite face of the rule to that already described. This line is numbered from the left-hand towards the right 1. 2. 3. 31. 32. which is at the right-hand end; it is then continued from the left-hand end of the other edge of the rule 32. 4. 5. 6. 7. 8. 9. 10. At 17.15 is a brass pin marked WG. signifying the circular gauge-point for wine gallons. At 18.95 is a brass pin marked AG for ale-gallons. At 46.37 MS signifies the square gauge point for malt bushels. At 52.32 MR signifies the round, or circular gauge-point for malt bushels. The line D on this rule is of the same nature as the line marked D on the *Carpenter's Rule,* described in chap. XI. page 52. The line A and the two slides B are used together, for performing multiplication, division, proportion, &c. and the line D, and the same slides B, are used together for extracting the square root, &c.

6thly, The other two slides belonging to this rule are marked C, and are divided in the same manner, and used together, like the slides B. *The back of the first radius or slide,* marked C, is divided, next the edge, into inches, and numbered from the left-hand towards the right 1. 2. 3. 4. &c. and these inches are again divided into ten equal parts. The second line is marked *spheroid,* and is numbered from the left-hand towards the right 1. 2. 3. 4. 5. 6. 7. The third line is marked *second variety,* and is numbered 1. 2. 3. 4. 5. 6. These lines are used, with the scale of inches, for finding a mean diameter, see prob. X. following.

The third and fourth variety are omitted on this slide, and with good reason, for it is very probable that there never was a cask made resembling either of these forms. *The back of the second radius, or slide,* marked C, contains several factors for reducing goods of one denomination to their equivalent value in those of another. Thus | X to VI 6. | signifies that to reduce *strong* beer at 8s. per barrel to *small* beer at 1s. 4d. you must multiply by 6. | VI. to X. 17. | signifies that to reduce small beer at 1s. 4d. per barrel to strong beer at 8s. per barrel, you must multiply by .17. | C 4s. to X. .27. | signifies that 27 is the multiplier for reducing cider at 4s. per barrel to strong beer at 8s. &c.

7thly, The two slides C, just described, are always used together, with the lines on the rule marked *seg. st.* or SS, segments standing; and *seg. ly* or SL, segments, lying, for ullaging casks. The former of these lines is numbered 1. 2. 3. 4. 5. 6. 7. 8, which stands at the right-hand end; it then goes on from the left-hand on the other edge 8. 9. 10, &c. to 100; the latter is numbered in a similar manner, 1. 2. 3. 4, which stands at the right-hand end; it then goes on from the left-hand on the other edge 4. 5. 6. to 100, &c.

PROBLEM I.

To find the several Multipliers, Divisors, and Gauge-points, belonging to the several Measures now used in England.

For Square figures, the following multipliers and divisors are to be used:

282)1.0000(.003546 multiplier for ale gallons.

231)1.0000(.004329 multiplier for wine gallons.

268.8)1.000(.0037202 multiplier for malt gallons.

2150.42)1.000(.00046502 multiplier for malt bushels.

227)1.000(.00405 multiplier for mash-tun gallons.

So, if the solid inches in any vessel be multiplied by the said multipliers, the product will be the gallons in the respective measures; or dividing by the divisors 282, 231, or 268.8; the quotient will likewise be gallons.

NOTE. That 282 solid inches is a gallon of ale or beer measure; 231 solid inches is a gallon of wine measure; 268.8 solid inches is a gallon, and 2150.42 solid inches is a bushel of malt, or corn measure.

For circular areas, the following multipliers and divisors are to be used:

282).785398(.002785 multiplier for ale gallons. +

231).785398(.003399 multiplier for wine gallons. +

.785398)282.(359.05 divisor for ale gallons.

.785398)231.(294.12 divisor for wine gallons.

.785398)2150.42(2738 divisor for corn bushels.

The square root of the divisor is the gauge-point.

The gauge-point for squares in	Ale measure, is	16.79
	Wine measure, is	15.19
	Malt bushel, is	46.37
The gauge-point for circular figures in	Ale measure, is	18.95
	Wine measure, is	17.15
	Malt bushel, is	52.32

And thus the numbers in the following table were calculated.

A TABLE of Multipliers, Divisors, and Gauge-points, for Squares and Circles.

Note. *The Areas, &c. are all in inches.*	*Multipliers for*		*Divisors for*		*Gauge-points for*	
	Squares	*Circles*	*Squares*	*Circles*	*Squares*	*Circles*
The side or diam. 1	1.	.785398		1.27324	1.	1.128
A superficial foot	.006944	.005454	144.	183.44	12.	13.54
A solid foot - -	.000578	.000454	1728.	2200.16	41.57	46.91
Ale gallon - - -	.003546	.002785	282.	359.05	16.79	18.95
Wine gallon -	.004329	.003399	231.	294.12	15.19	17.15
Malt, or corn bushel	.000465	.000365	2150.42	2738.	46.37	52.32
Malt gallon -	.003720	.002920	268.8	342.24	16.39	18.5
Mash-tun gallon	.004405	.00346	227.	289.	15.1	17.07
A lb. of H. soap cold	.036845	.028939	27.14	34.56	5.21	5.88
A lb. of hot soap	.035714	.028050	28.	35.65	5.29	5.97
A lb. of green soap	.038956	.0306	25.67	32.68	5.06	5.72
A lb. of W. S. soap	.039123	.030731	25.56	32.54	5.05	5.7
A lb. of tallow net	.031844	.025101	31.4	39.98	5.6	6.32
A lb. of G. starch	.028736	.022565	34.8	44.32	5.9	6.66
A lb. of dry starch	.024843	.019491	40.3	51.3	6.35	7.16
A lb. of flint glass	.094697	.074405	10.56	13.44	3.25	3.69
A lb. of W. glass	.071123	.05586	14.06	17.9	3.74	4.32
A lb. of G. glass	.082102	.064516	12.18	15.5	3.48	3.94

NOTE. It often happens in the practice of gauging, that when the one given number is set to the gauge-point on the sliding rule, the other given number will fall off the rule; hence in many cases it will be necessary to find a second, or new gauge-point.— The second gauge-points are the square roots of 10 times the divisors in the above table. Thus for squares,

the new gauge-point for ale is 53.10, for wine 48.06, for malt-bushels 14.66; and for circles, the new gauge-point for ale is 52.92, for wine 54.22, and for malt-bushels 16.54.

By the Sliding Rule.

Set 1 on B, to the old gauge-point on D, and against the other 1 on B, is the new gauge-point on D.

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plane Figure, whether triangular, quadrangular, or multangular.

To resolve this problem, you must, by chap. I. part II. find the area in inches, and then bring it to gallons, by dividing that area in inches by the proper divisor, *viz.* by 282 for ale, or by 231 for wine; or else by multiplying by .003546 for ale, or by .004329 for wine; and the quotient or product will be the area.

EXAMPLES.

1. Suppose a back or cooler in the form of a parallelogram, 250 inches in length, and 84.5 inches in breadth; what is the area in ale or wine gallons?

Multiply 250 by 84.5, and the product is 21125, the area in inches, which divide by 282, and the quotient is 74.9 gallons of ale; or multiplied by .003546, the product is 74.90925 gallons, nearly the same; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.45 gallons of wine.

Note. The areas of all plane figures, in gauging, are expressed in gallons; because there will be the

same number of solid inches in any vessel of one inch deep, as there are superficial inches in its base; so that what is called by gaugers a surface, or area, is in reality a solid of one inch in depth.

By the Sliding Rule.

on A.		on B.		on A.		on B.
As 282	:	84.5	::	250	:	74.9
As 231	:	84.5	::	250	:	91.45

2. If the side of a square be 40 inches, what is the area in ale gallons? *Ans.* 5.67 gallons.

3. The longest side of a parallelogram is 50 inches; the shortest side 30 inches; what is the area, in ale gollons? *Ans.* 5.32 gallons.

4. The side of a rhombus is 40 inches, and its perpendicular breadth 37 inches; what is its area in ale gallons? *Ans.* 5.25 gallons.

5. The length of a rhomboides is 48 inches, and its perpendicular breadth 32 inches; what is its area in wine gallons? *Ans.* 6.65 gallons.

6. The base of a triangle is 60 inches, and its perpendicular 23.5 inches; required its area in ale gallons? *Ans.* 2.5 gallons.

7. The diagonal of a trapezium is 60 inches, and the two perpendiculars from the opposite angles 15 and 27 inches; what is its area in ale gallons?

Ans. 4.47 gallons.

8. The side of a pentagon is 50 inches, what is its area in ale gallons? *Ans.* 15.25 gallons.

9. The side of a hexagon is 64 inches, and the perpendicular from the centre to the middle of one of the sides 55.42 inches; required its area in ale, wine gallons, and malt bushels?

Ans. { 37.73 ale gallons.
46.06 wine gallons.
4.94 malt bushels.

Y

B Cooke
July 27 1819

PROBLEM III.

The Diameter of a Circle being given in inches, to find the Area of it in Ale or Wine Gallons, &c.

If the square of the diameter be multiplied by .002785 for ale, or by .003399 for wine; or if it be divided by 359.05 for ale, or by 294.12 for wine, the products or quotients will be the respective ale or wine gallons: for any other denomination, use the proper multiplier or divisor in the table.

EXAMPLES.

1. Suppose the diameter of the circle be 32.6 inches; what will be the area in ale and wine gallons?

The square of 32.6 is 1062.76.

Then 359.05)1062.76(2.2592 area in ale gallons.
And 294.12)1062.76(3.6133 area in wine gallons.
Or $1062.76 \times .002785 = 2.9598$ ale gallons.
And $1062.76 \times .0034 = 3.6133$ wine gallons.

By the Sliding Rule.

	on D.		on B.		on D.		on B.
As	18.95	:	1	: :	32.6	:	2.96
As	17.15	:	1	: :	32.6	:	3.61

The two first terms are the circular gauge-points; see the table.

2. If the diameter of a circle be 45 inches, what is its area in ale gallons? *Ans.* 5.64 gallons.

3. If the diameter of a circle be 68 inches; required its area in ale and wine gallons, and malt bushels?

Ans. { 12.87 ale gallons.
15.72 wine gallons.
1.68 malt bushels.

PROBLEM IV.

The Transverse, or Longer Diameter, and the Conjugate, or Shorter Diameter, of an Ellipsis being given, to find its Area in Ale or Wine gallons.

If the rectangle or product of the two diameters, that is, of the length and breadth of an ellipsis, be divided by 359.05, or multiplied by .002785 for ale, or divided by 294.12, or multiplied by .0034 for wine, the quotient or product will be the ale or wine gallons required. And for any other denomination use the proper divisor or multiplier.

EXAMPLES.

1. Suppose the longer diameter be 81.4 inches, and the shorter diameter be 54.6 inches; what will be the area of that ellipsis in ale and wine gallons?

Multiply 81.4 by 54.6, and the product is 4444.44; then,

359.05)4444.44(12.38 area in ale gallons.
294.12)4444.44(15.11 area in wine gallons.
Or $4444.44 \times .002785 = 12.38$ ale gallons.
And $4444.44 \times .0034 = 15.11$ wine gallons.

By the Sliding Rule.

	on A.		on B.		on A		on B.
As	359	:	81.4	: :	54.6	:	12.4 ale gallons.
As	294	:	81.4	: :	54.6	:	15.1 wine gallons.

2. The transverse diameter of an ellipsis is 72 inches, conjugate 50 inches; what is its area in ale gallons?

Ans. 10 gallons.

3. If the transverse diameter of an ellipsis be 70 inches, conjugate 50 inches; what is its area in ale and wine gallons, and malt bushels?

$$\textit{Ans.} \begin{cases} 9.74 \text{ ale gallons.} \\ 11.90 \text{ wine gallons.} \\ 1.27 \text{ malt bushels.} \end{cases}$$

PROBLEM V.

To find the Content in Ale or Wine Gallons of any Prism, whatsoever form its base is of.

First, find its solid content in inches (by sect. I. II. III. of chap. II. part II.) then divide that content in inches by 282 for ale, or by 231 for wine; the respective quotients will be the content in ale or wine gallons.

Otherwise, you may find the content of a prism, by finding the area of its base in gallons (by problem II of this chapter) and multiply that area by the vessel's height, or depth within, the product will be its content in gallons.

EXAMPLES.

1. Suppose a vessel, whose base is a right-angled parallelogram, its length being 49.3 inches, its breadth 36.5 inches, and the depth of the vessel is 42.6 inches; the content in ale and wine gallons is required?

The length, breadth, and depth, being multiplied continually, the product is 76656.57; which divided by 282, the quotient is 271.83 ale gallons: and divided by 231, the quotient is 331.84 wine gallons: and by dividing by 2152.42, such a cistern will be found to hold 35.65 bushels of corn.

By the Sliding Rule.

on B. on D. on B. on D.
As the length : length : : breadth : mean proportion
on D. on B. on D. on B.
As sq. gauge p. : depth : : mean proportion : content
on B. on D. on B. on D.
viz. 49.3 : 49.3 : : 36.5 : 42.42
on D. on B.

$$\left.\begin{array}{r}16.79\\15.19\\46.37\end{array}\right\} \begin{array}{c}\text{on B.}\\ : 42.6\end{array} \;::\; \begin{array}{c}\text{on D.}\\ 42.42 :\end{array} \left\{\begin{array}{l}271.83 \text{ ale gal.}\\331.84 \text{ wine gal.}\\35.65 \text{ malt bush.}\end{array}\right.$$

2. Each side of the square base of a vessel is 40 inches, and its depth 10 inches; what is the content in ale gallons? *Ans.* 56.7 gallons.

3. The length of a rectangular parallelopipedon is 72 inches, breadth 33 inches, and depth 82 inches; required the content in ale and wine gallons, and malt bushels?

$$\textit{Ans.} \left\{\begin{array}{l}690.89 \text{ ale gallons.}\\843.42 \text{ wine gallons.}\\90.61 \text{ malt bushels.}\end{array}\right.$$

4. The diameter of a cylindrical vessel is 32 inches, the internal depth 45.5 inches; required its content in ale and wine gallons, and malt bushels?

$$\textit{Ans.} \left\{\begin{array}{l}129.78 \text{ ale gallons.}\\158.47 \text{ wine gallons.}\\17.01 \text{ malt bushels.}\end{array}\right.$$

PROBLEM VI.

To find the Content of any Vessel whose ends are Squares, or Rectangles of any dimensions.

RULE.

Multiply the sum of the lengths of the two ends by the sum of their breadths, to which add the areas of the two ends; this sum multiplied by one-sixth of the

depth will give the solidity in cubic inches; which divided by 282.231, or 2150.42 will give the content in ale gallons, wine gallons, or malt bushels.

EXAMPLES.

1. Suppose a vessel, whose bases are parallelograms: the length of the greater is 100 inches and its breadth 70 inches; the length of the less base 80, and its breadth 56, and the depth of the vessel 42 inches; the content in ale and wine gallons is required?

$\left.\begin{array}{l}180=100+80\text{, sum of the lengths}\\126=70+56\text{, sum of the breadths}\end{array}\right\}$ of the two ends.

22680 product.
7000 area of the greater base $=100\times70$.
4480 area of the less base $=80\times56$.

34160 sum.
7 one-sixth of the depth.

239120 solidity in cubic inches.

282)239120(847.94 ale gallons.
231)239120(1035.15 wine gallons.

By the Sliding Rule.

Find a mean proportional (83.66) between the length and breadth at the greater end, and a mean proportional (66.93) between the length and breadth at the less end; the sum of these is 150.59, twice a mean proportional between the length and breadth in the middle. Then

on D.		on B.		on D.		on B.
16.79	:	$\frac{42}{6}$	::	83.66	:	173.7
			::	66.93	:	111.2
			::	150.59	:	563.
						847.9 A. G.

The content in wine gallons may be found by using the wine gauge-point.

2. Each side of the bottom of a vessel, in the form of the frustum of a square pyramid, is 27 inches, each side at the top 13.8 inches, and the depth 21 inches; required the content in ale gallons?

Ans. 32.07 gallons.

3. There is a tun, whose parallel ends are rectangles, the length and breadth of the top 36 and 32 inches; the length and breadth at the bottom 48 and 40 inches, and the depth 60 inches: required the content of the tun in ale gallons, wine gallons, and malt bushels?

Ans. $\begin{cases} \text{323.41 ale gallons.} \\ \text{394.80 wine gallons.} \\ \text{42.4 malt bushels.} \end{cases}$

PROBLEM VII.

To find the Content of a Vessel, whose bases are parallel and circular, it being the Frustum of a Cone.

RULE.

To three times the product of the two diameters, add the square of their difference, multiply the sum by one-third of the depth, and divide the product by the proper circular divisor, *viz.* 359.05 for ale, 294.12 for wine, and 2738 for malt bushels, &c. and the quotient will be the content accordingly.

EXAMPLES.

1. Suppose the greater diameter 80 inches, and the less diameter 71 inches, and the depth 34 inches, the content in ale and wine gallons is required?

80 greater diameter.	71 less diameter.
71 less diameter.	80 greater diam.
9 difference.	5680 product.
9	3
81 square of the diff.	17040
	81 square of diff.
	3)17121 *sum.*
359.05)194031(540.42 A. G.	
294.12)194038(659.72 W. G.	5707
	34 depth.
	194038 product.

In the above example, instead of multiplying the *sum* by one-third of the depth, one-third of the sum is multiplied by the whole depth, which amounts to the same.

By the Sliding Rule.

Find a mean proportional between the two diameters: Thus,

on B.		on D.		on B.		on D.
80	:	80	::	71	:	75.36; again

Set 18.95 on D to $11\frac{1}{3}$ (one-third of the depth) on B.
And against 71 on D stands the content on B 159.09
against 75.36 on D stands the content on B 179.22
against 80 on D stands the content on B 202.00

Sum is the whole content 540.31

In the same manner the content in wine gallons may be found, using the wine gauge-point.

2. The greater diameter of a conical frustum is 38 inches, the less diameter 20.2, and depth 21 inches; required the content in ale gallons?

Ans. 51.07 gallons.

3. The top diameter of a conical frustum is 22 inches, bottom diameter 40 inches, and the depth 60 inches; required the content in ale gallons, wine gallons, and malt bushels?

Ans. $\begin{cases} 165.12 \text{ ale gallons.} \\ 201.55 \text{ wine gallons.} \\ 21.63 \text{ malt bushels.} \end{cases}$

PROBLEM VIII.

To Gauge and Inch a Tun in the form of the Frustum of a Cone, and to make an allowance for the drip or fall.

The inching of a tun, or vessel, is finding how much liquor it will hold at every inch of its depth.

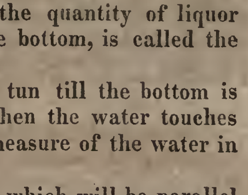
When a vessel does not stand even, or with its base parallel to the horizon, the quantity of liquor mDn, which will just cover the bottom, is called the drip or fall of the tun.

Let water be poured into a tun till the bottom is just covered, which will be when the water touches the point m, and suppose the measure of the water in this case to be 30.92 gallons.

Find the horizontal line FB, which will be parallel to $m\,n$, the surface of the liquor; the line FB will represent the surface of the liquor when the vessel is full.

Find the perpendicular depth $o\,n$ to the surface of the liquor, which suppose to be 26 inches.

Take mean diameters parallel to FB and $m\,n$ at every 6 or 10 inches depth. Suppose the first mean diameter $a\,a$ at 5 inches from FB to be 83.6 inches; the second mean diameter $b\,b$ at 15 inches from FB=78.7 inches and the third mean diameter $c\,c$ at 23 inches from FB=74.5 inches.



Find the areas correspondent to each of these diameters, as in the third column of the following table; multiply these areas by their respective depths and you will have the fourth column. Lastly, these contents being brought into barrels, &c. allowing 24 gallons to a barrel, and $8\frac{1}{2}$ gallons to a firkin, will give the remaining columns.

Depths.	Diam.	Area.	Content. in gallons.	Content. B.	F.	G.
$oP=10$	83.6	19.465	194.65	5	2	7.65
$PS=10$	78.7	17.250	172.50	5	0	2.50
$Sn=6$	74.5	15.458	92.75	2	2	7.75
whole 26	Cont. of the drip.		30.92	0	3	5.42
	Whole content.		490.82	14	1	6.32

Now to find the content at every inch of the depth, reduce the first area 19.465 into barrels, &c. and it will be 2 F. 2.465 G. which subtracted from the whole content 14 B. 1 F. 6.32 G. leaves 13 B. 3 F. 3.8555 G. the content when 1 inch is dry; and thus continue subtracting the first area from the several remainders until 10 inches are dry, when there will remain 8 B. 2 F. 7.17 G.—Then take the second area 17.25, having first brought it into firkins, &c. and subtract it from the last remainder, and thus continue to do till you have 20 inches dry, when there will remain 3 B. 2 F. 4.67 G.—Lastly, take the third area 15.458, having first brought it into firkins, &c. and subtract it from the area when 20 inches are dry; and thus proceed till you have 26 inches dry, and there will remain, if you have made no mistake, 3 F. 5.42 G. the quantity of liquor contained in the drip or fall.

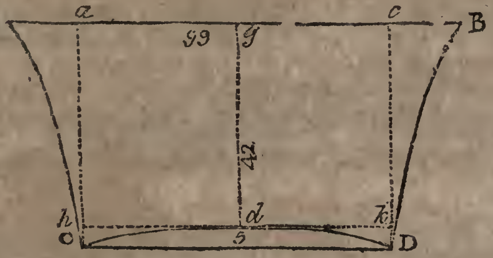
The point F may be marked as a constant dipping place, and the several contents entered into a dimension book. Then to find how much liquor there is in this fixed vessel, at any future time, take the depth of the liquor, and against that depth in your table you will find the content.

PROBLEM IX.

To gauge a Copper.

Let ABCD be a small copper to be gauged.

Take a small cord of packthread, make one end fast at A, and extend the other to the opposite side of the copper at B, where make it fast. Then with some convenient instrument take the nearest distance from the deepest place in the copper, to the thread, as aC, which suppose to be 47 inches.



In like manner, set the end of the instrument or rule upon the top of the crown at d, and take the nearest distance to the thread, as dg, which suppose 42 inches: this subtracted from aC, 47, the remainder 5 is the altitude of the crown.

To find CD, the diameter of the bottom of the crown.

Measure AB, the diameter of the top, which admit to be 99 inches; then hold a thread so as a plummet at the end thereof may hang just over C, by which means you will find the distance A*a*. Do the like on the other side: so will you find also the distance, which suppose 17.5 inches each; add these two together, and subtract their sum (*viz.* 35) from 99, and the remainder is 64 inches, equal to CD, the diameter at the bottom of the crown. The diameter *hk*, which touches the top of the crown, may be found, by measuring, to be 65 inches.

Now to find the content of the copper from the crown upwards, that is the part AB*kh*, the depth *gd* being 42 inches, you may take the diameter in the middle of every 6 inches of the depth, which suppose to be as in the second column of the following table, the numbers in the third column are the respective areas in ale gallons, found by prob. III. the fourth column shews the content of every 6 inches; all which being added together, the sum will be the content of that part, AB*kh*; that is, so much as it will hold after the crown is covered.

Now, if the crown be taken for the segment of a sphere, the content (by the latter part of sect. XI. p. 74.) will be found to be 28.75 gallons.

But may be more readily found, very near the truth, thus:

The diameter CD was found to be 64, and the area to this diameter is 11.408; this multiplied by half the crown's altitude, *viz.* by 2.5, gives 28.52 gallons, the content of the crown.

The content of the part *hk*DC is 57.935 gallons; from which subtract the content of the crown, 28.52, and the remainder is 29.415 gallons, and so much liquor will just cover the crown.

Parts of the depth.	*Diam.*	*Areas.*	*Content of every six inches.*
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	8[illegible].056
6	66.	12.1310	72.791
The sum - - - - - - - - - -			765.4[illegible]1
To just cover the crown - - -			29.415
The whole content - - - - - -			794.866

The contents in the last column may be brought into firkins and barrels, and then the content at every inch in depth, as in the VIIIth problem, may be found and entered into the dimension book.

PROBLEM X.

To compute the Content of any close Cask.

In order to perform this difficult part of gauging, the three following dimensions of the cask must be truly taken :

Viz. $\left\{\begin{array}{l}\text{The bung-diameter,}\\ \text{The head-diameter,}\\ \text{The length of the cask,}\end{array}\right\}$ within the cask.

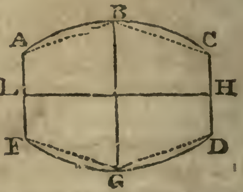
In taking these dimensions, it must be carefully observed,

1st, That the bung-hole be in the middle of the cask ; also, that the bung-stave, and the stave op-

posite to the bung-hole, are both regular and even within.

2dly, That the heads of the casks are equal, and truly circular; if so, the distance between the inside of the chimb to the outside of the opposite stave will be the head-diameter within the cask, very near.

The diameters and length of one cask may be equal to those of another, and yet one of those casks may contain several gallons more than the other.



As for instance, the figure ABCDF is supposed to represent a cask: then it is plain, that if the outward curve lines, ABC, and FGD, are the bounds or staves of the cask, it will hold more than if the inner dotted lines were the bounds, or staves; and yet the bung-diameter BG, and head-diameters CD and AF, and the length LH, are the same in both those casks.

Whence it appears, that no *one* general rule can be given, by which all sorts of casks may be gauged; and therefore gaugers usually suppose every cask to be in some of these forms:

1st, The middle zone or frustum of a spheroid; see page 190.

2dly, The middle frustum of a parabolic spindle; see page 198.

3dly, The lower frustums of two equal parabolic conoids; see page 193.

4thly, The lower frustums of two equal cones, such as the figure represented by the dotted lines above.

5thly, The contents of casks are sometimes found by having a mean diameter between the bung and head given; see the rule at page 200.

To find the Content of a Cask by the mean Diameter.

RULE.

Multiply the difference between the head and bung-diameters, when it is less than 6 inches, by .68 for the first variety; by .62 for the second variety; by .55 for the third; and by .5 for the fourth. Or, when the difference between the head and bung-diameters exceeds 6 inches, multiply that difference by .7 for the first variety; .64 for the second; .57 for the third; and .52 for the fourth. Add this product to the head diameter, and that sum will be a mean diameter. Square the mean diameter, and multiply that square by the length of the cask; this product multiplied, or divided, by the proper multiplier, or divisor, in the table, Prob. I. will give the content.

By the Sliding Rule.

Find the difference between the bung and head-diameter, on the inside of the slide marked C, and opposite thereto is, for each variety,* a number to be added to the head-diameter, for the mean diameter required. Then, as the gauge-point on D, is to the length on B: so is the mean diameter on D to the content on B.

---

* It has been noticed before, that only the first and second variety of casks are placed on the rule, which is described at the beginning of this chapter. The contents of casks are generally found by the sliding rule, for which reason the rules for finding the content by the pen, are given in such a manner as to agree therewith. The above methods of finding a mean diameter are not strictly true. See a dissertation on this subject, in Moss's Gauging, sect. X.

EXAMPLES.

1. Suppose the bung-diameter of a cask to be 32 inches, the head-diameter 24 inches, and the length 40 inches; the content in ale gallons, for each variety, is required?

1. For the spheroid, or first variety.

Bung-diameter	32
Head-diameter	24
Difference	8
Multiplier	.7
	5.6
Head-diam.	24.
Mean-diam.	29.6

Mean-diameter	29.6
	29.6
	1776
	2664
	592
	876.16 square.
	40 length.
	359.05)35046.40(97.6 gal.
	273190
	218550
	3120

In a similar manner you will find the content for the second variety to be 94.46 ale gallons, for the third variety 90.87 ale gallons; and for the fourth variety 88.34 ale gallons.

By the Sliding Rule.

For the first variety against 8 (the difference between the bung and head diameter) in the line of inches, you will find 5.6 on the line marked spheroid,

which added to the head-diameter 24, gives the mean-diameter 29.6. Then

on D.	on B.	on D.	on B.
18.95 :	40 ::	29.6 mean-diam.	: 97.6

In the same manner the contents for the other varieties may be found, by using their respective mean-diameters, without removing the slider.

2. Suppose the bung-diameter of a cask to be 26.5 inches, head-diameter 23 inches, and length 28.3 inches; the content in ale gallons, for each variety, is required?

Ans.
- 50.8 for the first variety.
- 50.0 for the second variety.
- 48.8 for the third variety.
- 48.28 for the fourth variety.

To find the Contents of any Cask.

GENERAL RULE.

Add into one sum
- 39 times the square of the bung-diameter,
- 25 times the square of the head-diameter, and
- 26 times the product of those diameters;

multiply the sum by the length of the cask, and the product by the number .00034; then this last product divided by 9 will give the wine gallons, and divided by 11 will give the ale gallons.

Dr. Hutton's Dict. Vol. I. page 528.

A General TABLE *for finding the Content of any Cask by the Sliding Rule.*

Q.	W. G.	A. G.	Q.	W. G.	A. G.	Q.	W. G.	A. G.
.50	21.31	23.55	.67	19.78	21.95	.84	18.36	20.29
.51	21.22	23.45	.68	19.69	21.76	.85	18.28	20.20
.52	21.13	23.35	.69	19.60	21.66	.86	18.20	20.12
.53	21.04	23.24	.70	19.51	21.56	.87	18.13	20.03
.54	20.95	23.14	.71	19.43	21.47	.88	18.05	19.95
.55	20.85	23.04	.72	19.35	21.37	.89	17.97	19.86
.56	20.76	22.94	.73	19.26	21.28	.90	17.89	19.77
.57	20.67	22.84	.74	19.18	21.18	.91	17.81	19.68
.58	20.57	22.74	.75	19.09	21.09	.92	17.74	19.60
.59	20.48	22.64	.76	19.01	21.01	.93	17.67	19.52
.60	20.39	22.54	.77	18.93	20.92	.94	17.59	19.44
.61	20.30	22.45	.78	18.85	20.83	.95	17.51	19.36
.62	20.21	22.35	.79	18.77	20.74	.96	17.44	19.27
.63	20.13	22.25	.80	18.69	20.65	.97	17.37	19.18
.64	20.04	22.15	.81	18.61	20.56	.98	17.30	19.10
.65	19.95	22.05	.82	18.53	20.47	.99	17.22	19.02
.66	19.87	21.86	.83	18.44	20.38	.100	17.15	18.95

The above table of gauge-points was calculated by Mr. *John Lowry*, an officer in the excise, and an ingenious mathematician. See the *Mathematical* and *Philosophical Repository*, page 119, &c.

The use of the Table.

Divide the head-diameter by the bung-diameter, to two places of decimals; find the quotient in the column marked Q, and against it stand the wine and ale gauge-points. Then,

As the gauge-point on D : the length of the cask on B : : the bung-diameter on D : the content on B.

EXAMPLES.

1. The head-diameter of a cask is 34.8 inches, the bung-diameter 44.8 inches, and the length of the cask

54 inches ; required the content in ale and wine gallons ?

Bung-diam. 44.8	Head-diam. 34.8	Bung-diam. 44.8
44.8	34.8	Head-diam. 34.8
3584	2784	3584
1792	1392	1792
1792	1044	1344
2007.04	1211.04 prod.	1559.04
39	25	26
1806336	605520	935424
602112	242208	311808
78274.56	30276.00	40535.04
30276.		
40535.04		

	8050622.4
149085.6 sum.	.00034
54 length.	
	322024896
5963424	241518672
7454280	
	11)2737.211616
8050622.4	
	248.8374 ale gallons.
	304.1346 wine galls.

By the Sliding Rule.

The quotient arising by dividing the head-diameter by the bung-diameter is .77, and the wine and ale gauge-points are 18.93 and 20.92.

$\left.\begin{matrix}18.93\\20.92\end{matrix}\right\}$ on D : 54 on B : : 44.8 on D : $\left\{\begin{matrix}\text{304 W. G. on B.}\\\text{248.A.G. —}\end{matrix}\right.$

2. The head-diameter of a cask is 24.5 inches; bung-diameter 31.5, and length 42 inches; required its content in ale and wine gallons?

Ans. { 95.766 ale gallons. 117.047 wine gallons.

The content by the sliding rule is exactly the same.

3. Required the content of a cask in ale and wine gallons, whose head-diameter is 24 inches, bung-diameter 32 inches, and length 40 inches?

Ans. { 112.28 wine gallons. 91.86 ale gallons.

The content by the sliding rule is the same.

4. The bung-diameter of a cask is 48 inches, head-diameter 35.8 inches, and the length 55 inches; required the content in ale and wine gallons?

Ans. { 283.178 ale gallons. 346.106 wine gallons.

The content is the same by the sliding rule.

5. The head-diameter of a cask is 28.2 inches, bung-diameter 33.8 inches, and length 48 inches; required the content in ale and wine gallons?

Ans. { 132.367 ale gallons. 161.782 wine gallons.

The content by the sliding rule is the same.

PROBLEM XI.

Of the Ullage of Casks.

The ullage of a cask is what it contains when only partly filled, and is considered in two positions, *viz.* standing on its end, or lying on one side.

To Ullage a lying Cask.

RULE.

Divide the wet inches by the bung-diameter: find the quotient in the column *height*, in the table at the end of this chapter; take out the corresponding *area seg.* multiply this area by the content of the cask, and that product by 1.27324; or divide it by .7854, the last product, or quotient, will give the ullage nearly.

By the Sliding Rule.

Set the bung-diameter on C to 100 on the line marked *seg. ly.* or SL, *viz.* segments lying: then look for the wet inches on C, and observe what number stands against it on the segments, which call a fourth number. Then set 100 on A to the content of the cask upon B, and against the fourth number, before found, on A is the quantity of liquor in the cask on B.

EXAMPLES.

1. Supposing the bung-diameter of a lying cask to be 32 inches, its content 97.6 ale gallons; required the ullage for 19 wet inches?

```
        1.000 whole diameter.
32)19.000(.594 height.
                              .7854 whole area.
        0.406 height.  Area seg. .299255
                              -------
                              .486145 rem.
              The content       97.6
                              -------
                              2916870
                             3403015
                            4375305
                            -------
              .7854)47.4477520(64.4 gal.
```

Note. Because, in this example, the quotient of the wet inches, divided by the bung-diameter, ex-

ceeds the heights in the table, it is necessary to subtract it from the whole diameter, &c.

By the Sliding Rule.

	on C.		on SL.		on C.		on B.
As	32	:	100	: :	19	:	62.5 fourth number.
	on A.		on B.		on A.		on SL.
As	100	:	97.6	: :	62.5	:	61 gallons, *Ans.*

2. The bung-diameter of a lying cask is 25 inches, the content 48.3 ale gallons and the wet inches 11; required the quantity of liquor in the cask?

Ans. { 20.46 by calculation.
19.9 by the sliding rule.

3. The bung-diameter of a lying cask is 32 inches, its content 91.6 gallons, and the wet inches 8; required the quantity of liquor in the cask?

Ans. { 17.9 gallons, by calculation.
16. gallons, by the sliding rule.

To Ullage a standing Cask.

RULE.

Multiply the difference between the squares of the bung and head diameters, by the square of the distance of the liquor's surface from the middle of the cask; and divide the product by the square of half the length of the cask, subtract one-third of the quotient from the square of the bung-diameter, and multiply the remainder by the distance of the liquor's surface from the middle of the cask.

The last product divided by 359.05 for ale, or 294.12 for wine, will give the quantity of liquor above, or under half the content of the cask, according as the wet inches exceed, or fall short of half the length of the cask.

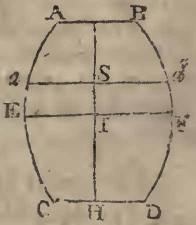
By the Sliding Rule.

Set the length of the cask on C to 100 on the line marked *seg. st.* or SS, *viz.* segments standing; then look for the wet inches on C, and observe what number stands against it on the segments, which call a fourth number.

Then, set 100 on A to the content of the cask upon B, and against the fourth number, before found, on A is the quantity of liquor in the cask on B.

EXAMPLES.

1. Suppose in the annexed cask, whose content is 97.6 ale gallons, that the bung diameter EF be 32 inches, head-diameter AB 24 inches, length 40 inches, and the wet inches SH 26; what quantity of ale is contained in the cask?



$$\begin{array}{r}32=EF\\32\\\hline 64\\96\\\hline 1024\\576\\\hline 448 \text{ diff.}\\36\\\hline 2688\\1344\\\hline 400)\,16128\\\hline 3)40.32 \text{ quot.}\\\hline 13.44\end{array}\qquad\begin{array}{r}24=AB\\24\\\hline 96\\48\\\hline 576\end{array}\qquad\begin{array}{l}26=SH\\20=IH\\\hline 6 \text{ diff.}=SI\\6\\\hline 36 \text{ square of } SI\end{array}$$

$$\begin{array}{r}20=IH\\20\\\hline 400=\overline{IH}^2\end{array}$$

$$\begin{array}{r}1024 \text{ square of } EF\\13.44\\\hline 1010.56\\6=SI\\\hline\end{array}$$

359.05)6063.36(16.887 gallons,

above half the content of the cask, *viz.* in *ad* EF. To which add 48.8 gallons, half the content of the cask, and the sum is 65.687, the quantity of the liquor in the cask.

By the Sliding Rule.

on C.	on SS.	on C.	on SS.
As 40 :	100 : :	26 :	66.1 fourth number.
on A.	on B	on A.	on B.
As 100 :	71.6 : :	66.1 :	64.6 gallons, *Ans.*

2. The bung-diameter of a standing cask is 35 inches, head-diameter 28.7, length 40, wet inches 30; content in ale gallons 121.5, in wine gallons 148.5; required the content of the ullage in ale and wine gallons?

Ans. { 93.93 ale gallons. 114.76 wine gallons.

The answer by the sliding rule is the same.

3. The bung-diameter of a standing cask is 26.5 inches, head-diameter 23 inches, length 28.3, wet inches 11; and the content in ale gallons 48.3; required the ullage?

Ans. { 18.01 gallons, by calculation. 18.3 by the sliding rule.

The use of the following table in gauging has been shewn at the beginning of the 11th problem; but its chief use is for finding the area of a segment of a circle. Thus,

Divide the height of the segment, by the diameter of that circle of which it is the segment, to three places of decimals; find the quotient in the column *height*, take out the corresponding *Area Seg.* which multiply by the square of the aforesaid diameter, and the product will be the area of the segment required.

If the quotient of the height by the diameter be greater than 15, subtract it from an unit, and find the

area seg. corresponding to the remainder; which subtract from .7854 for the *area seg.*

If the quotient of the height by the diameter do not terminate in three figures, find the *area seg.* answering to the first three decimals of the quotient; subtract it from the next greater *area seg.* multiply the remainder by the fractional part of the quotient, and add the product to the first *area seg.* taken out.

EXAMPLES.

1. Required the area of the segment of a circle, whose height is 3.25; the diameter of the circle being 50?

50)3.25(.065 quotient, or tabular height.

The tabular segment is .021659, which multiplied by 2500, the square of the diameter, gives 54.1475, the area required.

2. Required the area of the segment of a circle, whose height is 46.75, and diameter of the circle 50?

	1.000	
50)46.75)	.935 quotient.	Whole area .7854
	.065 tab. height.	*Area seg.* .021659
	Remains, tabular *area seg.*	.763741

which multiplied by 2500 gives 1909.3525, the area required.

3. Required the area of the segment of a circle whose height is 2, and diameter of the circle 52?

52)2.000(.038$\frac{24}{52}$=.038$\frac{6}{13}$=quotient.

The *area seg.* answering to .038 is .009763; the next greater *area seg.* is .010148; the difference is .000385, $\frac{6}{13}$ of which is .000177, which added to .009763 gives .009940, the *area seg.* corresponding to .038$\frac{6}{13}$: hence .009940×square of 52=26.87776 answer.

A TABLE OF THE AREAS

OF THE

SEGMENTS OF A CIRCLE,

Whose Diameter is Unity, and supposed to be divided into 1000 equal parts.

Heig.	*Area Seg.*	*Heig.*	*Area Seg.*	*Heig.*	*Area Seg.*
.001	.000042	.036	.009008	.071	.024680
.002	.000119	.037	.009385	.072	.025195
.003	.000219	.038	.009763	.073	.025714
.004	.000337	.039	.010148	.074	.026236
.005	.000470	.040	.010537	.075	.026761
.006	.000618	.041	.010931	.076	.027289
.007	.000770	.042	.011330	.077	.027821
.008	.000950	.043	.011730	.078	.028356
.009	.001135	.044	.012142	.079	.028894
.010	.001320	.045	.012554	.080	.0294[illegible]
.011	.001533	.046	.012971	.081	.029979
.012	.001746	.047	.013392	.082	.030526
.013	.001968	.048	.013818	.083	.031076
.014	.002199	.049	.014247	.084	.031629
.015	.002438	.050	.014681	.085	.032186
.016	.002685	.051	.014119	.086	.032745
.017	.002940	.052	.015561	.087	.033307
.018	.003202	.053	.016007	.088	.033872
.019	.003471	.054	.016457	.089	.034441
.020	.003748	.055	.016921	.090	.035011
.021	.004031	.056	.017369	.091	.035585
.022	.004322	.057	.017831	.092	.036162
.023	.004618	.058	.018296	.093	.036741
.024	.004921	.059	.018766	.094	.037323
.025	.005230	.060	.019239	.095	.037909
.026	.005546	.061	.019716	.096	.038496
.027	.005867	.062	.020196	.097	.039087
.028	.006194	.063	.020691	.098	.039680
.029	.006527	.064	.021168	.099	.040276
.030	.006865	.065	.021650	.100	.040875
.031	.006909	.066	.022154	.101	.041476
.032	.007558	.067	.022652	.102	.042080
.033	.007913	.068	.023154	.103	.042687
.034	.008275	.069	.023659	.104	.043296
.035	.008638	.070	.024168	.105	.043908

Heig.	Area Seg.	Heig.	Area Seg.	Heig.	Area Seg.
.106	.044522	.152	.075306	.198	.110226
.107	.045139	.153	.076026	.199	.111024
.108	.045759	.154	.076747	.200	.111823
.109	.046381	.155	.077469	.201	.112624
.110	.047005	.156	.078194	.202	.113426
.111	.047632	.157	.078921	.203	.114230
.112	.048262	.158	.079649	.204	.115035
.113	.048894	.159	.080380	.205	.115842
.114	.049528	.160	.081112	.206	.116650
.115	.050165	.161	.081846	.207	.117460
.116	.050804	.162	.082582	.208	.118271
.117	.051446	.163	.083320	.209	.119083
.118	.052090	.164	.084059	.210	.119897
.119	.052736	.165	.084801	.211	.120712
.120	.053385	.166	.085544	.212	.121529
.121	.054036	.167	.086289	.213	.122347
.122	.054689	.168	.087036	.214	.123167
.123	.055345	.169	.087785	.215	.123988
.124	.056003	.170	.088535	.216	.124810
.125	.056663	.171	.089287	.217	.125634
.126	.057226	.172	.090041	.218	.126459
.127	.057991	.173	.090797	.219	.127285
.128	.058658	.174	.091554	.220	.128113
.129	.059327	.175	.092313	.221	.128942
.130	.059999	.176	.093074	.222	.129773
.131	.060672	.177	.093836	.223	.130605
.132	.061348	.178	.094601	.224	.131438
.133	.062026	.179	.095366	.225	.132272
.134	.062707	.180	.096134	.226	.133108
.135	.063389	.181	.096903	.227	.133945
.136	.064074	.182	.097674	.228	.134784
.137	.064760	.183	.098447	.229	.135624
.138	.065449	.184	.099221	.230	.136465
.139	.066140	.185	.099997	.231	.137307
.140	.066833	.186	.100774	.232	.138150
.141	.067528	.187	.101553	.233	.138995
.142	.068225	.188	.102334	.234	.139841
.143	.068924	.189	.103116	.235	.140688
.144	.069625	.190	.103900	.236	.141537
.145	.070328	.191	.104685	.237	.142387
.146	.071033	.192	.105472	.238	.143238
.147	.071741	.193	.106261	.239	.144091
.148	.072450	.194	.107051	.240	.144944
.149	.073161	.195	.107842	.241	.145799
.150	.073874	.196	.108636	.242	.146655
.151	.074589	.197	.109430	.243	.147512

Heig.	Area Seg.	Heig.	Area Seg.	Heig.	Area Seg.
.244	.148371	.290	.189047	.336	.231689
.245	.149230	.291	.189055	.337	.232634
.246	.150091	.292	.190804	.338	.233580
.247	.150953	.293	.191775	.339	.234546
.248	.151816	.294	.192684	.340	.235473
.249	.152680	.295	.193596	.341	.236421
.250	.153546	.296	.194509	.342	.237369
.251	.154412	.297	.195422	.343	.238318
.252	.155280	.298	.196337	.344	.239268
.253	.156149	.299	.197252	.345	.240218
.254	.157019	.300	.198168	.346	.241169
.255	.157890	.301	.199085	.347	.242121
.256	.158762	.302	.200003	.348	.243074
.257	.159636	.303	.200922	.349	.244026
.258	.160510	.304	.201841	.350	.244980
.259	.161386	.305	.202761	.351	.245934
.260	.162263	.306	.203683	.352	.246889
.261	.163140	.307	.204605	.353	.247845
.262	.164019	.308	.205527	.354	.248801
.263	.164899	.309	.206451	.355	.249757
.264	.165780	.310	.207376	.356	.250715
.265	.166663	.311	.208301	.357	.251673
.266	.167546	.312	.209227	.358	.252631
.267	.168430	.313	.210154	.359	.253590
.268	.169315	.314	.211082	.360	.254550
.269	.170202	.315	.212011	.361	.255510
.270	.171089	.316	.212940	.362	.256471
.271	.171978	.317	.213871	.363	.257433
.272	.172867	.318	.214802	.364	.258395
.273	.173758	.319	.215733	.365	.259357
.274	.174649	.320	.216666	.366	.260320
.275	.175542	.321	.217599	.367	.261284
.276	.176435	.322	.218533	.368	.262248
.277	.177330	.323	.219468	.369	.263213
.278	.178225	.324	.220404	.370	.264178
.279	.179122	.325	.221340	.371	.265144
.280	.180019	.326	.222277	.372	.266111
.281	.180918	.327	.223215	.373	.267078
.282	.181817	.328	.224154	.374	.268045
.283	.182718	.329	.225093	.375	.269013
.284	.183619	.330	.226033	.376	.269982
.285	.184521	.331	.226974	.377	.270951
.286	.185425	.332	.227915	.378	.271920
.287	.186329	.333	.228858	.379	.272890
.288	.187234	.334	.229801	.380	.273861
.289	.188140	.335	.230745	.381	.274832

Heig.	*Area Seg.*	*Heig.*	*Area Seg.*	*Heig.*	*Area Seg.*
.382	.275803	.422	.315016	.462	.344736
.383	.276775	.423	.316004	.463	.355732
.384	.277748	.424	.316992	.464	.356730
.385	.278721	.425	.317981	.465	.357727
.386	.279694	.426	.318970	.466	.358725
.387	.280668	.427	.319959	.467	.359723
.388	.281642	.428	.320948	.468	.360721
.389	.282617	.429	.321938	.469	.361719
.390	.283592	.430	.322928	.470	.362717
.391	.284568	.431	.323918	.471	.363715
.392	.285544	.432	.324909	.472	.364713
.393	.286521	.433	.325900	.473	.365712
.394	.287498	.434	.326892	.474	.366710
.395	.288476	.435	.327882	.475	.367709
.396	.289453	.436	.328874	.476	.368708
.397	.290432	.437	.329866	.477	.369707
.398	.291411	.438	.330858	.478	.370706
.399	.292390	.439	.331850	.479	.371705
.400	.293369	.440	.332843	.480	.372704
.401	.294349	.441	.333836	.481	.373703
.302	.295330	.442	.334829	.482	.374702
.403	.296311	.443	.335822	.483	.375702
.404	.297292	.444	.336816	.484	.376702
.405	.298273	.445	.337810	.485	.377701
.406	.299255	.446	.338824	.486	.378701
.407	.300238	.447	.339798	.487	.379700
.408	.301220	.448	.340793	.488	.380700
.409	.302203	.449	.341787	.489	.381699
.410	.303187	.450	.342782	.490	.382699
.411	.304171	.351	.343777	.491	.383699
.412	.305155	.452	.344772	.492	.384699
.413	.306140	.453	.345768	.493	.385699
.414	.307125	.454	.346764	.494	.386699
.415	.308110	.455	.347759	.495	.387699
.416	.309095	.456	.348755	.496	.388699
.417	.310081	.457	.349752	.497	.389699
.418	.311068	.458	.350748	.498	.390699
.419	.312054	.459	.351745	.499	.391699
.420	.313041	.460	.352742	.500	.392699
.421	.314029	.461	.353739		

CHAPTER VI.

Surveying.

SURVEYING is the art of measuring, planning, and finding the superficial content, of any field, or parcel of land. In this kind of measuring, the area or superficial content is always expressed in acres; or acres, roods, and perches; and the lengths of all lines, in the field, or parcel of land, are measured with a chain, such as is described at page 78.

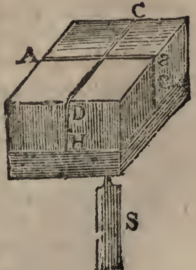
A line, or distance on the ground, is thus measured. Having procured 10 small arrows or iron rods, to stick in the ground at the end of each chain; also some station-staves, or long poles with coloured flags, to set up at the end of a station-line, or in the angles of a field; two persons take hold of the chain, one at each end; the foremost, for the sake of distinction, is called the leader, the hindermost the follower.

A station-staff is set up in the direction of the line to be measured, if there be not some object, as a tree, a house, &c. in that direction.

The leader takes the 10 arrows in the left hand, and one end of the chain, by the ring, in his right hand, and proceeds towards the station-staff, or other object. The follower stands at the beginning of the line, holding the other end of the chain, by the ring, till it is stretched straight, and laid, or held level, by the leader, whom he directs, by waving his hand to the right or left, till he see him exactly in a line with the object towards which they are measuring. The leader then sticks an arrow upright in the ground, as a mark for the follower to come to, and proceeds forward another

chain, at the end of which he is directed, as before, by the follower; or he may now, and at the end of every other chain, direct himself, by moving to the right or left, till the follower and the object measured from, be in one straight line. Having stuck down an arrow, as before, the follower takes up the arrow which the leader first stuck down. And thus they proceed till all the 10 arrows are employed, or in the hands of the follower, and the leader, without an arrow, is arrived at the end of the eleventh chain length. The follower then sends or carries the 10 arrows to the leader, who puts one of them down at his end of the chain, and proceeds with the other nine and the chain as before. The arrows are thus changed from the one to the other, till the whole line is finished, if it exceed 10 chains; and the number of changes shews how many times 10 chains the line contains. Thus, if the whole line measures 36 chains 45 links, or 3645 links, the arrows have been changed three times, the follower will have 5 arrows in his hand, the leader 4, and it will be 45 links from the last arrow, to be taken up by the follower, to the end of the line.

Of the Surveying-Cross, or Cross-Staff.



The surveying cross consists of two pair of sights at right angles to each other: these sights are sometimes pierced out in the circumference of a thick tube of brass; and sometimes the cross-staff consists of four sights strongly fixed upon a brass cross, and when used is screwed on a staff, having a sharp point to stick in the ground. The accuracy of the cross-staff depends on the sights being exactly at right

angles to each other. A cross-staff may be easily made by any carpenter. Thus, take a piece of beech or box, ADBC, of four or five inches in breadth, and three or four inches in depth, and upon ADBC draw two lines, AB and CD, crossing each other at right angles. Then with a fine saw make two slits, ABG and CDH, of about two inches in depth; fix this piece of wood upon a staff S, of about 4 and a half or 5 feet in length, pointed at one end, so that it may easily stick into the ground.

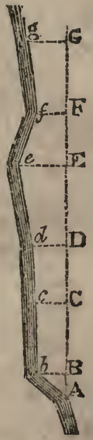
PROBLEM I.

To Measure Off-Sets with a Chain and Cross-Staff.

Let A*bcdefg* be a crooked hedge, river, or brook, &c. and AG a base line. First, begin at the point A, and measure towards G: when you come to B, where you judge a perpendicular must be erected, place the cross-staff in the line AG in such a position that both G and A may be seen through two of the sights, looking forward towards G, and backward towards A.—Then look along one of the cross-sights, and if it point directly to the corner, or bend at *b*, the cross-staff is placed right; otherwise move backward or forward along AG till the cross-sights do point to *b*, and measure B*b*, which set down in links: proceed thus till you have taken all the off-sets, as in the following

FIELD-BOOK.

Off-sets left.	Base line AG, or, ⊙, stations.	Off-sets right.
	⊙A.	
62	45	——
84	220	——
70	340	——
98	510	——
57	634	——
91	785	——
		——



To lay down the Plan.

Draw the line AG of an indefinite length. Then, by a diagonal scale, such as described at page 48, set off AB equal to 45 links, draw Bb perpendicular to AG, and equal to 62 links. Next set off AC equal to 220 links, or 2 chains, 20 links; draw Cc perpendicular to AG, and equal to 34 links: then set off AD equal to 340 links; or 3 chains 40 links, and make Dd equal to 70 links: proceed thus till you have completed the figure.

To cast up the Content.

ABb must be measured as a triangle; BCcb, CDdc, DEed, &c. must be measured as trapezoids; see page 93. Some authors direct you to add all the perpendiculars Bb, Cc, &c. together, and divide their sum by the number of them, then multiply the quotient by the length AG; but this method is always erroneous, except the off-sets Bb, Cc, &c. be equally distant from each other.

CALCULATION.

AB=45	AC=220	AD=340	AE=510	AF=634	AG=784
B*b*=62	AB=45	AC=220	AD=340	AE=510	AF=633
90	BC=175	CD=120	DE=170	EF=124	FG=151
270					
	B*b*=62	C*c*=84	D*d*=70	E*e*=98	F*f*=57
2790	C*c*=84	D*d*=70	E*e*=98	F*f*=57	G*g*=91
	Sum 146	Sum 154	Sum 168	Sum 155	Sum 148
	Bc=175	CD=120	DE=170	EF=124	FG=151
Prod.	25550	18480	28560	19220	22348

2790 = double area of AB*b*.
25550 = double area of BC*cb*.
18480 = double area of CD*dc*.
28560 = double area of DE*ed*.
19220 = double area of EF*fe*.
22348 = double area of FG*gf*.

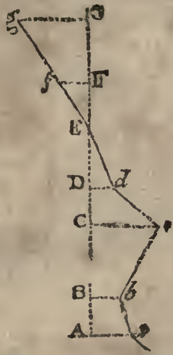
2)116948 = double area of the whole in sq. links.

58474 = area in square links.

.58474 = area in acres=0 A. 2 R. 13.5584 P.

2. Required the plan and content of part of a field, from the following field-book.

Off-sets left.	*Base line* AG.	*Off-sets right.*
——	⊙A.	100
——	100	60
——	250	150
——	325	46
——	450	Cross Hedge.
60	550	——
150	700	——



The figure must be laid down, and the content calculated as in the first example. Thus you will find the area of the part E *d c b a* A to be 1 R. 13. 4. P. and of the part E *f g* G to be 30 perches; so that the whole is 2 roods 3.4 perches.

4.36

PROBLEM II.

To measure a Field in the form of a Trapezium.

Set up station-staves, or long poles, at the corners A, B, C. Then begin at D, and measure along the diagonal DB, in a right line, till you come to the place of the perpendicular AF, which will be known by looking backward and forward through the sights of the cross-staff, as before directed. Make a mark at F, and measure the perpendicular AF; then proceed from F towards B, and when you come to E, the place where the second perpendicular will fall, make a mark, and measure the perpendicular EC: lastly, continue your measure from E to B. You may either draw a rough plan of the field by the eye, and write the length of the diagonal and perpendiculars on it, which some

writers recommend as the best method; or set them down in a field-book, thus:

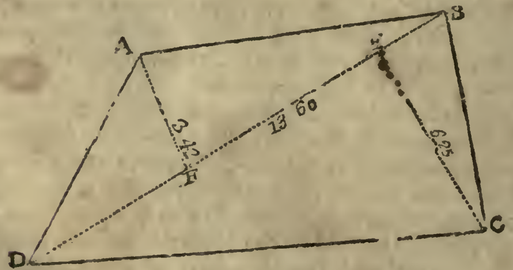
Off-sets left.	*Station, or base line.*	*Off-sets right.*
——	⊙ D	——
342	600	——
——	1190	625
——	1360	——

To lay down the Plan.

Draw the station-line DB equal to 1360 links, or 13.60 chains; from D set off DF equal to 600 links, or 6 chains; draw AF perpendicular to DB, and equal to 342 links, or 3 chains 42 links: make DE equal to 1190 links, and at E erect the perpendicular EC equal to 625 links. Join DA, AB, BC and DC, and the field is constructed.

CALCULATION.

This field being a trapezium, its content must be found as directed in section VI. chapter I. part II.



AF=342 links.
EC=625

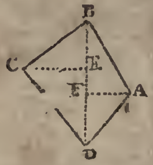
Sum 967
680 half the diagonal DB.

77360
5802

6.57560 area in acres=6 acres 2 rods
12.096 perches.

2. Required the plan and content of a field from the following field-book.

Off-sets left.	*Stations, or base lines.*	*Off-sets right.*
	⊙ D.	
——	214	210
306	362	——
——	592	——



Answer. The content is 1 acre 2 roods, 4.3776 per.

NOTE. In either of the above figures, if the sides DC, CB, BA, AD, and the diagonal DB had been measured with the chain, the figures might have been planned, and their contents found by the rule, page 84, without using a cross-staff, or measuring the perpendiculars AF and CE.

Some, who pretend to measure land, always measure round a four-sided field, and cast up the content by adding every two opposite sides together, and taking half their sum; and then multiply these half sums into each other. It may be necessary to inform the learner, that this method is very erroneous, and ought *never* to be practised.

PROBLEM III.

To measure a four-sided Field with crooked Hedges.

Set up staves, or poles, at the corners D, C, B, as before directed. Then begin at A, and measure from AB, noting all the necessary off-sets, and in this manner go round the field, then measure the diagonal AC.

FIELD-BOOK.

Off-sets left.	*Stations, or base lines.*	*Off-sets right.*
	⊙ A.	
——	300	100
——	400	130
——	550	80
——	750=AB	——
	⊙ B.	
——	200	200
——	700	150
——	1500=BC	——
	⊙ C.	
——	300	200
——	500	100
——	700	150
——	900	50
——	1000=CD	——
	⊙ D.	
150	200	——
200	400	——
100	700	——
——	800=DA	——
Diagonal AC=1294.		

CALCULATION.

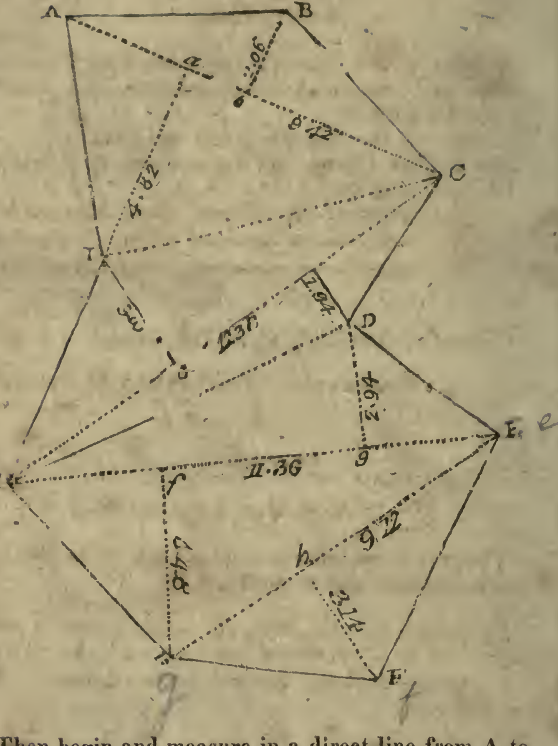
	Acres.
The sides of the triangle ABC are 750, 1500, and 1294 links, and its content, by the rule, page 84 - - - - - - -	4.85239
The sides of the triangle ACD are 1000, 800, and 1294 links, and its content, by the rule, page 84 - - - - - - -	3.99907
Content of the off-sets along AB=	0.50250
Ditto ——————— along BC=	1.67500
Ditto ——————— along CD=	1.07500
Sum=	12.10396
Content of the off-sets along DA, deduct	1.00000
Remains the content of the field=	11.10396

PROBLEM IV.

How to Measure an Irregular Field.

The way to measure irregular land, is to divide it into trapeziums and triangles, thus:

First, look over the field, and set up marks at every angle, and by those marks you may see where to have a trapezium, as ABCI in the following figure.



Then begin and measure in a direct line from A towards C; but when you come to *a*, set up your cross, and try whether you be in a square to I (as is before directed): and then measure the perpendicular *a* I, which is 482 links; then measure forward again toward C, but when you come to *b*, set up your cross, and try whether you be in the place where the perpendicular will fall; then measure the perpendicular *b* B,

which is 206 links; then continue your measure to C, and you will find the whole diagonal 942 links.

Then proceed to measure the trapezium CDHI, beginning at C, and measuring along the diagonal line towards H: but when you come to *d*, set up your cross, and try if you be in the place where the perpendicular will fall: measure the perpendicular *d*D, which is 146 links; and then measure forward till you come to c, and there, with your cross, try if you be right in the place where the perpendicular will fall, and measure the perpendicular *c*I, which is 3 chains; and from *c* continue your measure to H, and you will find the whole diagonal 1236 links.

Then proceed to measure the trapezium HGED, beginning at H, and measuring along the diagonal line towards E; but when you come to *f*, try with your cross if you be in the place where the perpendicular will fall; and measure the perpendicular *f*G, which is 448 links; then continue on your measure from *f* till you come to *g*, and there try if you be in a square with the perpendicular *g*D; and measure the said perpendicular, which is 294 links; then measure on from *g* to E, and you will find the whole diagonal to be 1144 links.

Then measure the triangle EFG, beginning at E, and measuring along the base EG, till you come to *h*, and there with your cross try if you be in the place where the perpendicular will fall; and measure the perpendicular *h*F, which is 314 links, continue your measure to G, and you will find the whole base to be 912 links; so you have finished your whole field.

But to draw a plan of the field, it would be necessary to measure a few more lines, or mark the points *a*, *b*, &c.

CALCULATION.

The area of the trapezium ABCI	=324048
The area of the trapezium CDHI	=275628
The area of the trapezium HGED	=420714
The area of the triangle EFG	=143184
The area of the whole -	1163574

Cut off five figures from the right-hand, and the result will be 11.63574 acres=11 acres 2 roods 21.7184 p.

2. Required the plan and content of an irregular field, from the following

FIELD-BOOK.

Off-sets left.	Station, or Base Line.	Off-sets right.
	⊙ F.	
380	240	——
——	380	480
553	735	265
——	940	600
140	1100	——
——	1410	435
——	1620	——

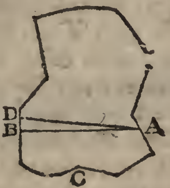
This field, when constructed, is exactly similar to the former one. The content of the off-sets must be found as in problem 1. Thus,

	Square Links:
The content of ABCDEF will be found	601000
The content of AIHGF ————	439390
The area of the whole -	1040390

Cut off five figures, and the result will be 10.40390 acres=10 A. 1 R. 24.624 P.

PROBLEM V.

To cut off from a Plan a given number of Acres, &c. by a line drawn from any point in the side of it.



Let A be the given point in the annexed plan, from which a line is to be drawn towards B, so as to cut off 5 acres 2 roods 14 perches. Draw AB, so as to cut off a quantity ABC, as near the quantity proposed as you can judge; and suppose the true quantity of ABC, when calculated to be only 4 A. 3 R. 20 P. which is 2 R. 34 P.=114 perches=71250 square links too little. Then measure AB, which suppose equal to 1234 links, by the half of which, *viz.* 617 links, let 71250 links be divided; the quotient, 115 links, will be the altitude of the triangle to be added, whose base is AB, therefore make BD=115 links, and draw AD, which will cut off the quantity required.

CHAPTER VII.

Practical Questions in Measuring.

Question 1. IF a pavement be 47 feet 9 inches long, and 18 feet 6 inches broad, I demand how many yards are contained in it? *Ans.* 98 yards 1 foot.

Quest. 2. There is a room, whose length is 21.5 feet, and the breadth 17.5 feet, which is to be paved with

stones, each 15 inches square; I demand how many such stones will pave it?

Ans. $167\frac{2}{9}$ stones.

Quest. 3. There is a room 109 feet 9 inches about, and 9 feet 3 inches high, which is all (except two windows, each 6 feet 6 inches high, and 5 feet 9 inches broad) to be hung with tapestry that is ell-broad; I desire to know how many yards will hang the said room?

From the content of the room, subtract the content of the windows, and divide the remainder by the square feet in a yard of tapestry, *viz.* $14\frac{1}{4}$ feet, and you will find the answer to be 83.59 yards.

Quest. 4. If the axis of a globe be 27.5 inches; I demand the content solid and superficial?

Ans. $\begin{cases} \text{10889.24375 inches} = \text{6.3 feet solid.} \\ \text{2375.835 inches} = \text{16.49 feet superficies.} \end{cases}$

Quest. 5. There is a segment of a globe, the diameter of whose base is 24 inches, and its altitude 19 inches; what is the content solid and superficial, including the area of its base?

Ans. $\begin{cases} \text{2785.552 inches the solidity.} \\ \text{1218.9408 inches the superficies.} \end{cases}$

The diameter of the whole sphere may be found as in page 118, to be 24.4 inches.

Quest 6. If a tree girt 18 feet 6 inches, and be 24 feet long, how many tons of timber are contained in that tree, using the customary method of measuring, and allowing 40 feet of timber to a ton?

Ans. 12 tons 33 feet 4 inches 6 parts.

Quest. 7. There is a cellar to be dug by the floor, the length of which is 33 feet 7 inches, and the breadth 18 feet 9 inches, and its depth is to be 5 feet 9 inches; I demand how many floors of earth are in that cellar?

Ans. 11 floors 56 feet 8 inches 5 parts.

NOTE. That 18 feet square and a foot deep is a floor of earth, that is, 324 solid feet.

Quest. 8. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 35 feet 6 inches, and the length 48 feet 9 inches; how many squares of tiling are contained in it? *Ans.* 17 squares 30 feet $7\frac{1}{2}$ inches.

Quest 9. There is a cone, the diameter at the base being 42 inches, and the perpendicular height 94 inches; and it is required to cut off two solid feet from the top end of it; I demand what length upon the perpendicular must be cut off?

First, find the solidity of the whole cone, 43410.6288 cubic inches, or 25.1219 feet, and the cube of the altitude 830584 inches. Then, since all similar solids are in proportion to each other as the cubes of their like parts,

	Feet.		Inches.		Feet.		Inches.
As	25.1219	:	830584	::	2	:	66124.297,

the cube of the altitude to be cut off, the cube-root of which is 40.436 isches, *Answer.*

Quest. 10. If a square piece of timber be 12 feet long, and if the side of the square of the greater base be 21 inches, and the side of the square of the lesser base be 3 inches; how far must I measure from the greater end, to cut off five solid feet?

You will find the length of the whole pyramid by the rule, page 118, to be 14 feet. Then find the solid

content of the whole pyramid $14.291\frac{2}{3}$ feet, from which deduct 5 feet, the remainder is $9.291\frac{2}{3}$ feet.

	Solidity.		Cube length.		Solidity.		Cube length.
As	$14.291\frac{2}{3}$	:	2744	::	$9.291\frac{2}{3}$	:	1784
Or	42.875	:	2744	::	27.875	:	1784 the

cube-root of which is 12.128 feet, which subtracted from the whole length, 14 feet, leaves 1.872 feet, the length of 5 solid feet at the greater end.

Quest. 11. Three men bought a grindstone of 40 inches diameter, which cost 20 shillings; of which sum the first man paid 9 shillings, the second 6 shillings, and the third 5 shillings: I demand how much of the stone each man must grind down, proportionable to the money he paid?

All circles are to each other as the squares of their diameters; and each man must grind away a surface proportionable to the money he paid.

$20 : 20\times20 :: 5 : 100$, the square-root of 10,
$20 : 20\times20 :: 6 : 120$

Sum 220, the square-root is 14.832397, the radius which two men must grind down from the centre, from which take 10, the radius which one man must grind down, there remains 4.832397, the breadth of the ring which the second man must grind away: from the whole radius 20 subtract 14.832397, the radius which two men must grind away; the remainder is 5.167603, the breadth of the ring the first man must grind away.

OR GEOMETRICALLY THUS:

First, upon the centre $\odot$ describe the circle ABCD, and cross it at right angles with the two diameters AB and CD: then divide the semidiameter A $\odot$ in proportion to 9s. 6s. and 5s. the several sums paid by the three men; *viz.* make AE 9, EF 6, and F $\odot$ 5: then divide EB into two equal parts in d, and upon d, as a centre, describe the semicircle E a B; divide FB into two equal parts in c, and upon c, as a centre, with the radius c F, describe the semicircle F b B; which will divide the semidiameter $\odot$ C into three such parts as the stone ought to be divided; and circles described through these points, will shew how much each man must grind for his share.

This construction is derived from the above method of calculation for $B\odot=20$, $\odot F=5$, and $\odot E=11$, by the property of the circle $B\odot\times\odot F=\overline{\odot b}^2$, and $B\odot\times\odot E=\overline{\odot a}^2$; also $\odot a-\odot b=ab$ and $\odot C-\odot a=aC$: hence $\odot b=10$, $ab=4.83$, and $aC=5.17$ nearly.

Quest. 12, A gard'ner had an upright cone,
Out of which should be cut him a rolling-stone,
The biggest that e'er it could make:
The mason he said, that there was a rule
For such sort of work, but he had a thick skull:
Now help him for pity's sake.

Ans. It must be cut at one-third part of the altitude.

NOTE. This is properly a question in fluxions, and not dependant upon any rule in this book. See *Simpson's Geometry*, on the *Maxima* and *Minima* of geometrical quantities, theorem XIX.

Quest. 13. There is a cistern, whose depth is seven tenths of the width, and the length is 6 times the depth, and the solid capacity is 367.5 feet; I demand the depth, width, and length, and how many bushels of corn it will hold?

First, you must find three numbers in proportion to the depth, width, and length thus: suppose the width 1, the depth will be .7, and the length 4.2; hence the solidity will be 2.94 feet. But solids are to each other as the cubes of their like parts, consequently

2.94 feet : 1 cube width :: 367.5 feet : 125 the cube of the real width, the cube-root of which is 5 feet, the width; hence the depth is 3.5 feet, and length 21 feet.

The content is 295 bushels 1 peck 4 pints.

Quest. 14. Suppose, sir, a bushel be exactly round,
Whose depth being measur'd, 8 inches is found;
If the breadth 18 inches and half you discover,
This bushel is legal all *England* over.
But a workman would make one of another frame;
Sev'n inch and a half must be the depth of the same;
Now, sir, of what length must the diameter be,
That it may with the former in measure agree?

Ans. 19.107 inches must be the diameter when the depth is $7\frac{1}{2}$ inches.

Quest. 15. In the midst of a meadow, well stored
with grass,
I took just an acre, to tether my ass:
How long must the cord be, that, feeding all round,
He may'nt graze less nor more than his acre of
ground? *Ans.* 117 feet 9 inches.

Quest. 16. A malster has a kiln, that is 16 feet 6 inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

Ans. The side of the new one must be 28 feet and near 7 inches, or 28.578 feet.

Quest. 17. If a round cistern be 26.3 inches diameter, and 52.5 inches deep; how many inches diameter must a cistern be to hold twice the quantity, the depth being the same? and how many ale gallons will each cistern hold?

Ans. The diameter of the greater cistern is 37.19 inches, and its content 202.275 gallons; hence the content of the less cistern must be 101.137 gallons.

Quest. 18. If the diameter of a cask at the bung be 32 inches, and at the head 25 inches, and the length 40 inches; how many ale gallons are contained therein?

Ans. 94.41 gallons, by the general rule, page 269.

Quest. 19. There is a stone, 20 inches long, 15 inches broad, and 8 inches thick, which weighs 217 pounds; I demand the length, breadth, and thickness of another of the same kind and shape, which weighs 1000 pounds?

The cube of 20, the length, is 8000. Then

217 : 8000 :: 1000 : 36866.3594, whose cube root is 33.282 inches, the length of the stone weighing 1000 pounds. Then say,

20 : 33.282 : : 15 : 24.961

20 : 33.282 : : 8 : 13.312

Ans. $\left\{\begin{array}{ll}\text{The length} & 33.282 \\ \text{The breadth} & 24.961 \\ \text{The thickness} & 13.312\end{array}\right\}$ inches.

Quest. 20. If an iron bullet, whose diameter is 4 inches, weighs 9 pounds; what will be the weight of another bullet (of the same metal) whose diameter is 9 inches? *Ans.* 102.515 pounds.

Quest. 21. There is a square pyramid of marble, each side of its base is 5 inches, and the height 15 inches, and its weight is 12 pounds and a quarter; I demand the weight of another like square pyramid, each side of whose base is 30 inches?

The cube of 5 is 125, and the cube of 30 is 27000. Then (by *Eucl.* XII. 12.)

lb. *lb.*

125 : 12.25 : : 27000 : 2646

Ans. The weight is 2646 pounds.

Quest. 22. There is a ball or globe of marble, whose diameter is 6 inches, and its weight 11 pounds; what will be the diameter of another globe of the same marble, that weighs 500 pounds? *Ans.* 21.4 inches.

Quest. 23. There is a frustum of a pyramid, whose bases are regular octagons; each side of the greater base is 21 inches, and each side of the less base is 9 inches, and its perpendicular length is 15 feet; I demand how many solid feet are contained in it?

Ans. 119.2 feet.

Quest. 24. There is a frustum of a cone, the diameter of the greater base is 36 inches, and the diameter of the less base is 20 inches, and the length or height is 215 inches; I demand the length and solid content of the whole cone, and also the solid content of the given frustum?

First, find the length of the whole cone, thus:

As 8, the difference between the radii of the two ends : 215, the length of the frustum : : 18, the radius of the greater end : 483.75 inches, the whole length of the cone.

The solidity of the whole cone is 94.98 feet.

The solidity of the frustum is 78.7 feet.

Quest. 25. If the top part of a cone contains 26171 solid inches, and 200 inches in length, and the lower frustum thereof contains 159610 solid inches; I demand the length of the whole cone, and the diameter of each base?

		Inches.
Ans.	The length of the whole cone	384.3766
	The diameter of the greater base	42.9671
	The diameter of the less base	22.3568

Quest. 26. There is a frustum of a cone, whose solid content is 20 feet, and its length 12 feet; and the greater diameter bears such proportion to the less as 5 to 2; I demand the diameters?

First, I find the content of a conical frustum, whose diameters are 5 and 2, and depth 12 feet, to be 122.5224 feet. Then, as 122.522 : 20 feet : : 25, square of the greater diameter : 4.080886, the square root of which is 2.02012 feet, the true greater diameter; and 5 : 2 : : 2.02012 : .80804 feet, the less diameter required.

Or, the two required diameters are 24.2414 inches and 9.6964 inches.

Quest. 27. There is a room of wainscot 129 feet 6 inches in circumference, and 16 feet 9 inches high (being girt over the mouldings;) there are two windows, each 7 feet 3 inches high, and the breadth of each, from cheek to cheek, 5 feet 6 inches; the breadth of the shutters of each is 4 feet 6 inches; the cheek-boards

and top and bottom-boards of each window, taken together, are 24 feet 6 inches, and their breadth 1 foot 9 inches; the door-case 7 feet high, and 3 feet 6 inches wide; the door 3 feet 3 inches wide; I demand how many yards of wainscot are contained in that room?

The content of the room	2169	1	6
The shutters, at work and half	97	10	6
The door, at work and half	34	1	6
The cheek-boards	85	9	0
The sum	2386	10	6
The window-lights and door-case deduct	104	3	0
	9)2282	7	6
	253	5	

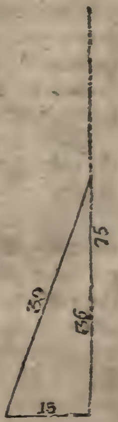
Ans. 253 yards 5 feet.

Quest. 28. There is a wall which contains 18225 cubic feet, and the height is 5 times the breadth, and the length 8 times the height; what is the length, breadth, and height?

Assume the breadth 1, then the height must be 5, and length 40; hence the solidity will be 200. Then say

200 : 1, cube of the breadth : : 18225 : 91.125, the cube of the real breadth; the cube-root of which is 4.5 the breadth; hence the height is 22.5, and the length 180 feet.

Quest. 29. There is a may-pole, whose top-end was broken off by a blast of wind, and, in falling, struck the ground at 15 feet distance from the bottom of the may-pole: the broken piece was 39 feet; now I demand the length of the may-pole?



From the square of 39, the length of the broken piece, subtract the square of 15; the square-root of the remainder is the piece standing; to which add the piece broken off, and you have the whole length. Thus you will find

The piece standing to be 36 feet.
The piece broken off is 39 feet.

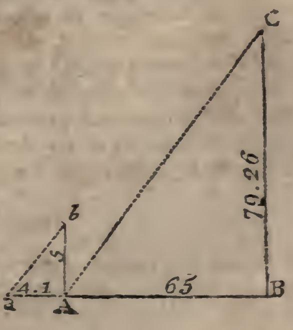
The whole length is 75

Quest. 30. A may-pole there was, whose height I would know,
The sun shining clear, strait to work I did go.
The length of the shadow, upon level ground,
Just sixty-five feet, when measur'd, I found;
A staff I had there, just five feet in length—
The length of its shadow was four feet one-tenth:
How high was the may-pole, I gladly would know?
And it is the thing you're desired to show.

Here A B represents the length of the shadow of the may-pole, and BC its height; aA the shadow of the staff, and Ab its height.

By similar triangles.

$aA : Ab :: AB : BC$.
The height BC will be found to be 79.26 feet.



Quest. 31. What will be the diameter of a globe, when the solidity and superficial content of it are equal?

Ans. 6: When the diameter of the globe is 1, the solidity is to the superficies, as 1 to 6.

Quest. 32. What will the axis of a globe be, when the solidity is in proportion to the superficies, as 18 to 8? *Ans.* 13.5.

Quest. 33. There are three grenado shells, of such capacity, that the second shell will just lie in the concavity of the first, and the third in the concavity of the second. The solidity of the metal of the first shell is equal to the number of solid inches which would fill its capacity: the solidity of the metal of the second shell is in proportion to the number of solid inches which would fill its concavity, as 7 is to 5; and the solidity of the metal of the third shell is in proportion to the number of solid inches which would fill its concavity, as 9 is to 4. The diameter of the first shell is 16 inches; required the diameters of the second and third shells, the thickness of each shell, and also the weight, supposing a solid inch of iron to weigh 4 ounces avoirdupois?

If the diameter of a globe be 1, its solidity will be .5236, and globes are in proportion to each other as the cubes of their diameter: hence

1 : .5236 :: cube of 16=4096 : 2144.6656, the half of which is 1072.3328, the solidity of the first shell, and the number of solid inches which would fill its concavity; which being divided by .5236, and the cube-root of the quotient extracted, will give 12.699, the internal diameter of the *first* shell; hence its thickness is 1.65 inches, and weight 268.08 pounds.

SECONDLY.

Since the second shell will just fill the concavity of the first, its external diameter must be equal to

12.699, and the solidity of its metal and concavity together, equal to 1072.3328 inches.

745 : 1072.3328 :: 5 : 446.80533, the solid inches which would fill the concavity of the second shell; which being divided by .5236, and the cube-root of the quotient extracted, will give 9.485 inches, the internal diameter of the second shell; hence its thickness is 1.607 inches, and weight 156.38 pounds.

THIRDLY.

Since the third shell will just fill the concavity of the second, its external diameter must be equal to 9.485 inches, and the solidity of its metal and concavity together, equal to 446.80533 inches.

9+4 : 446.80533 :: 4 : 137.47846, the solid inches which would fill the concavity of the third shell; which being divided by .5236, and the cube-root of the quotient extracted, will give 6.4034 inches, the internal diameter of the third shell; hence its thickness is 1.541 inch, and weight 77.33 pounds.

FINIS.

EXPLANATION OF THE CHARACTERS MADE USE OF IN THE FOREGOING WORK.

Charact.	*Names.*	*Significations.*
$+$	Plus, or more,	the sign of addition, as $2+4$ signifies that 2 and 4 are to be added together.
$-$	Minus, or less	the sign of subtraction, as $8-3$ signifies that 3 is to be subtracted from 8.
$\times$	multiplied into or by,	the sign of multiplication, as 7×5 signifies that 7 is to be multiplied into or by 5.
$\div$	divided by,	the sign of division, as $9\div3$ signifies that 9 is to be divided by 3; and $\frac{9}{3}$ or 3-9, signifies the same.
$=$	equal to	the sign of equality, as $9=9$ signifies that 9 is equal to 9, or $5+4-2=7$ signifies that 5, increased by 4 and diminished by 2 is equal to 7.
$: :: :$	Proportion,	as $2 : 4 :: 8 : 16$ signifies that 2 is to 4 as 8 is to 16.

m^2, m^3, signifies the square or cube of the letter m.

$\overline{TS}^2$ signifies the square of the line TS.

$\sqrt{\overline{aA}}$ signifies the square root of aA.

APPENDIX.

OF THE

WEIGHTS AND DIMENSIONS

OF

BALLS AND SHELLS.

THE weights of bodies, composed of the same substance, are proportional to their magnitudes; and the magnitudes of similar bodies are proportional to the cubes of their corresponding lineal dimensions; therefore, since all globes are similar bodies, the weights of those, which are formed of the same substance, must be proportional to the cubes of their diameters. It is well known, however, that different portions of metal, even from the same *casting*, will differ a small matter, in density; and also that a still greater difference is produced by different degrees of temperature: the theory here laid down, is therefore not to be understood as absolutely true, but sufficiently so for the purposes to which it is applied.

PROBLEM I.

Given the diameter of an Iron Ball, to find its weight.

An iron ball of 4 inches diameter, is known, from experiment, to weigh 9 lb. avoirdupoise; therefore, since the weights are as the cubes of the diamaters, it will be, as the cube of 4, (or 64) is to 9, so is the cube of

the given diameter, to the weight required. Hence, $\frac{9}{64}$ of the cube of the diameter will be the weight in lbs. But $\frac{9}{64}=\frac{1}{8}+\frac{1}{8}$ of $\frac{1}{8}$; which gives the following

RULE.

To $\frac{1}{8}$ of the cube of the diameter, add $\frac{1}{8}$ of that $\frac{1}{8}$; the sum will be the weight in lbs.

EXAMPLES.

1. Required the weight of an iron ball whose diameter is 6.4 inches.

The cube of 6.4 is	262.144
$\frac{1}{8}$ of which is - -	32.768
$\frac{1}{8}$ of $\frac{1}{8}$ is - - - -	4.096
Weight of the ball -	36.864 lb.

2. The diameter of an iron ball is 8 inches, what is its weight. *Ans.* 72 lb.

3. What is the weight of an iron ball, its diameter being 2.4 inches. *Ans.* 1 lb. 15.1 oz.

PROBLEM II.

To find the weight of a Leaden Ball.

A leaden ball of 3 inches diameter weighs 6 lb. Therefore as the cube of 3 is to 6; that is, as 9 to 2 so is the cube of the diameter to the weight of a leaden ball. Hence the following practical

RULE.

Take $\frac{2}{9}$ of the cube of the diameter of a leaden ball, for its weight in pounds.

EXAMPLES.

1. Required the weight of a leaden ball, whose diamter is 6.3 inches.

$\frac{2}{9}$ of $6.3\times6.3\times6.3=55.566$ lb. the weight required.

2. What is the weight of a leaden ball, 8.1 inches diameter? *Ans.* 118 lb.

3. The diameter of a leaden ball is 1.8 inches, what is its weight? *Ans.* 1 lb. 4.736 oz.

PROBLEM III.

Given the weight of an Iron Ball, to find its diameter.

The converse of prob. 1, will give this

RULE.

Multiply the given weight by $7\frac{1}{9}$, and the cube root of the product will be the diameter: or, to the constant logarithm 0.85194 add the logarithm of the weight in pounds, and $\frac{1}{3}$ of the sum will be the logarithm of the diameter, in inches.

EXAMPLES.

1. Required the diameter of a 42 lb. iron ball.

Constant log.	0.85194
Log. of 42	1.62325
	3)2.47519
Diam. 6.685, log.	0.82506

2. What is the diameter of an iron ball, weighing 18 lb. *Ans.* 5.04 inches.

3. An iron ball weighs 6 lb. what is its diameter. *Ans.* 3.494 inches.

PROBLEM IV.

The weight of a Leaden Ball being given, to find its diameter.

The converse of prob. 2, will give this

RULE.

Multiply the weight by $4\frac{1}{2}$, and the cube root of the product will be the diameter: or, to the constant logarithm 0.65321, add the logarithm of the weight, and $\frac{1}{3}$ of the sum will be the logarithm of the diameter.

EXAMPLES.

1. Required the diameter of a 64 lb. leaden ball.

Constant log.	0.65321
Log. of 64	1.80618
Diam. 6.605, log.	3)2.45939
	0.81979

2. What is the diameter of a leaden ball weighing 18 lbs? *Ans.* 4.327 inches.

3. A leaden ball weighs 9 lbs. what is its diameter? *Ans.* 3.434 inches.

PROBLEM V.

To find the weight of an Iron Shell.

RULE.

To $\frac{1}{8}$ of the difference of the cubes of the external and internal diameters, add its $\frac{1}{8}$, the sum will be the weight of the shell.

EXAMPLES.

1. Required the weight of an iron shell, the external diameter being 11, and the internal, 9 inches.

The cube of 11 is	1331
The cube of 9	729
Difference	602
$\frac{1}{8}$	$75\frac{8}{32}$
$\frac{1}{8}$ of $\frac{1}{8}$	$9\frac{13}{32}$
Weight of the shell	$84\frac{21}{32}$ lb.

2. To find the weight of a shell whose external diameter is 9.6 inches, and internal 7.2 inches.

Ans. 71.928 lb.

3. What is the weight of an iron shell, the outer diameter being 18 inches, and the inner 16.

Ans. $244\frac{1}{8}$ lb.

PROBLEM VI.

To find what weight of Powder will fill a given Shell.

RULE.

Divide the cube of the internal diameter, in inches, by 58, the quotient will be the lbs. of powder.

EXAMPLES.

1. What weight of powder will fill a shell, whose internal diameter is 9 inches.

$9 \times 9 \times 9 \div 58 = 12\frac{33}{58}$ lb. the quantity required.

2. What quantity of powder will fill a shell 7.2 inches in diameter. *Ans.* 6.435 lb.

3. Required the weight of powder to fill a shell whose inner diameter is 16 inches. *Ans.* $70\frac{18}{29}$ lb.

PROBLEM VII.

To find the diameter of a Shell to contain a given weight of Powder.

RULE.

Multiply the pounds of powder by 58; the cube root of the product will be the internal diameter, in inches.

EXAMPLES.

1. To find the diameter of a shell to contain 9 lb. of powder.

   $58 \times 9 = 522$; the cube-root of which is 8.05 inches nearly.

2. Required the diameter of a shell, to contain $47\frac{1}{3}$ lb. of powder. *Ans.* 14 inches, nearly.

3. To find the diameter of a shell, to contain 27 lb. of powder. *Ans.* [illegible] inches.

13.+

N. B. The two last rules were deduced from experiments on the powder manufactured for the government of the United States.

Answer 13+.

Ans. 13+

B